Accompanying Notes and Documentation on Development of DICE-2007 Model:

Notes on DICE-2007.delta.v8 as of September 21, 2007

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Preliminary

I. Description of the Accompanying Notes

These notes are intended to provide more detail on the derivation and calibration of the different components of the DICE-2007,delta.v8 model revision underlying the DICE model of September 5, 2007 underlying the DICE model. The current study, William Nordhaus, "The Challenge of Global Warming: Economic Models and Environmental Policy,"is dated September 11, 2007. These notes are necessarily incomplete because some of the steps involved detailed spreadsheets, but the major elements should be clear.

Where the entry is labeled "Lab notes," these are drawn from contemporaneous notes on the procedures. They have sometimes been edited but not been rewritten for the present Accompanying Notes. These lab notes will help users understand the reasoning underlying the estimates.

Cautionary note on the detailed estimates in these Accompanying Notes: The data in these notes are not published at the present stage because their quality is significantly less than the aggregated data in the published study. They are provided so that researchers can understand how the aggregate numbers were developed. The detailed estimates are often based on judgmental estimates, extrapolations from other estimates, or on less reliable source data.

II. Estimates of the backstop technology

A. Description

This was a major new element in DICE-2007. The mathematics of the backstop is straightforward. By solving the cost function for the derivative of cost w.r.t. emission, the marginal cost at miu = 1 is the backstop price.

The estimated level of the backstop technology was around $1170 per ton C in 2005, declining by a logistic function at an initial rate of 0.5 percent per year to an asymptotic cost of $585 per ton C. These are the cost of 100 percent displacement of carbon emissions.

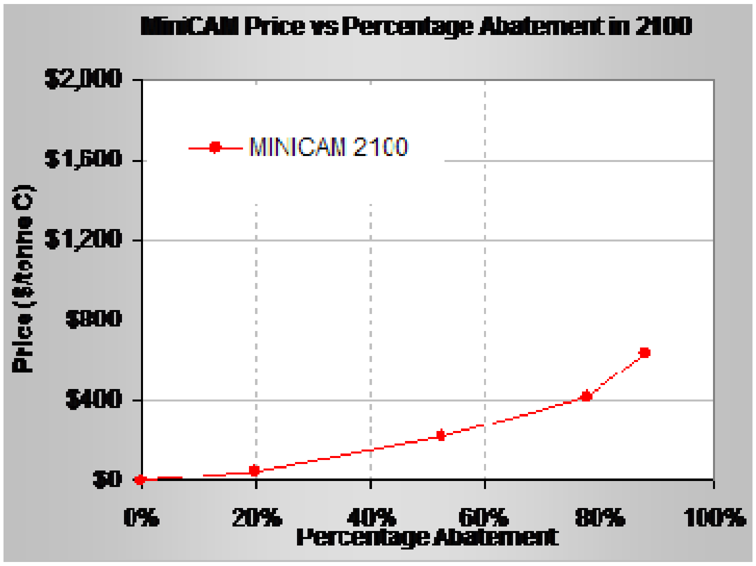
B. Lab Notes

For empirical estimates, we looked to three sources. First, the IPCC study of sequestration is shown in the following table. The results are as follows. The numbers are quite a bit lower than our backstop technologies. For electricity, including transportation and storage, the upper estimate is around $500 per ton C (for ocean storage). This is about half of our estimate. We can see the difference in the results from the following, which substitutes a backstop of $500 per ton C:

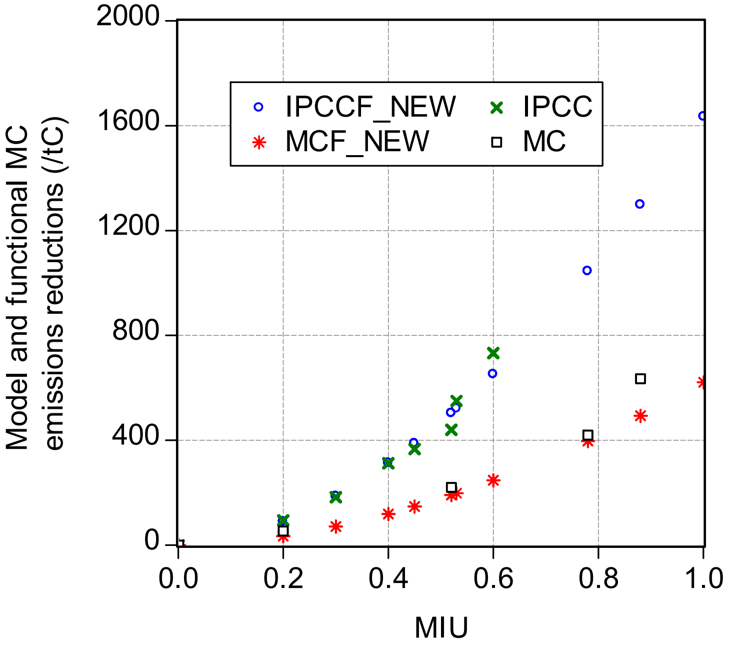
CCS system components	Cost of capture or injection (2002 $)				Remarks
	per ton CO2 (range)		per ton C (range)		
Capture from a coal- or gas-fired power plant	15	75	55	275	Net costs of captured CO2, compared to the same plant without capture.
Capture from hydrogen and ammonia production or gas processing	5.0	55	18	201	Applies to high-purity sources requiring simple drying and compression.
Capture from other industrial sources	25	115	92	421	Range reflects use of a number of different technologies and fuels.
Transportation	1.0	8.0	3.7	29	Per 250 km pipeline or shipping for mass flow rates of 5 (high end) to 40 (low end) MtCO2 yr-1.
Geological storage	0.5	8.0	1.8	29	Excluding potential revenues from EOR or ECBM.
Geological storage: monitoring and verification	0.1	0.3	0.4	1.1	This covers pre-injection, injection, and post-injection monitoring, and depends on the regulatory requirements.
Ocean storage	5.0	30	18	110	Including offshore transportation of 100-500 km, excluding monitoring and verification.
Mineral carbonation	50	100	183	366	Range for the best case

Source: *IPCC Special Report: Carbon Dioxide Capture and Storage,* Bert Metz, Ogunlade Davidson, Heleen de Coninck, Manuela Loos, and Leo Meyer, eds., Second, we looked at estimates provided by Jae Edmonds (personal communication, shown next. For this, we took the Edmonds estimates of marginal cost at high miu (personal communication, January 9, 2007, included at the end). Estimating Jae's function, we get the following. The estimated backstop for 2100 is also shown. The exponent is higher in this round (closer to DICE 1994). Based on this, we set exponent = 2.8 and 2100 backstop = 800.

We have adjusted the model with this introduction. We then define the base run as no controls for 150 years, then optimize after that. (Later, we adjusted that to 250 years.) The carbon tax with Jae's parameterization is 19.18 v. 19.86, this is slightly lower. This is probably because the initial cost is lower with this parameterization, and therefore the initial miu is lower.



Finally, we examined the preliminary compilation of estimates from the draft report for the FAR. A review of preliminary results from second draft FAR IPCC (SDFAR) indicates that the Edmonds estimates are relatively optimistic. The exponent is virtually identical the Edmonds runs (1.79+1), so the strong convexity seems to be realistic. The estimate of the cost of the backstop technology for 2050 is $1634 for the SDFAR and $621 for Edmonds for 2100. Note that few of the SDFAR scenarios go beyond 60 percent reduction, and the upper limit is $150 per t CO2 ($550 per t C). This further calibration suggests a great deal of uncertainty about the backstop technology cost and that the estimates here are consistent with the existing model estimates. The following graph shows the model and formulaic estimates for the two sources. MC is Edmonds, SDFAR is from the review. The results are in jae_calibration.wf1.


Model and functional MC
emissions reductions (/tC)
2000
1600
1200
800
400
0
0.0
0.2
0.4
0.6
0.8
1.0
MIU
IPCCF_NEW
IPCC
MCF_NEW
MC

III. Estimates of the parameters of the abatement cost function

a. General Description

The estimates of the abatement cost, or miu, function takes the abatement cost estimates described above and estimates a log-linear function. Since the data are marginal cost data, we can integrate this to get the cost function. The important finding is the great convexity of the function, with a well-determined exponent near 3 from all the data sets.

b. Lab notes

In early runs, we had limited the increase in miu at .25 per decade. This gave poor results in many of the constrained runs, and it was therefore dropped in the d.8 version. The only major difference is in the highly constrained runs.

Given the problems with the base run, it seems best to have that a variant of the optimal with the miu at the Hotelling or at zero for the first few periods. 150 years seems a reasonable guess. This also keeps cumulative C consumption well below the limit.

obs	MIU	MC	MCF	MCFLN
1	0.88	635	602.55	662.50
2	0.78	420	470.30	522.68
3	0.52	221	204.47	235.59
4	0.2	54	28.7185	36.028
5	0.001	0.001	0.00053	0.0010
6	1		783.508	851.71

```
Dependent Variable: MC
Method: Least Squares
Date: 01/11/07   Time: 15:10
Sample (adjusted): 1 5
Included observations: 5 after adjustments
Convergence achieved after 1 iteration
MC=C(1)*MIU^C(2)
=========================================================================
                        Coefficient     Std. Error    t-Statistic    Prob.
=========================================================================
```

```
            C(1)            783.5084     67.83342     11.55048     0.0014
            C(2)            2.054282     0.344458     5.963815     0.0094
==========================================================================
R-squared                   0.983807   Mean dependent var        266.0002
Adjusted R-squared          0.978409   S.D. dependent var        263.4395
S.E. of regression          38.70945   Akaike info criterion     10.43922
Sum squared resid           4495.263   Schwarz criterion         10.28299
Log likelihood             -24.09805   Durbin-Watson stat        2.675909
==========================================================================
```

```
Dependent Variable: LOG(MC)
Method: Least Squares
Date: 01/11/07   Time: 15:10
Sample (adjusted): 1 5
Included observations: 5 after adjustments
==========================================================================
          Variable        Coefficient    Std. Error   t-Statistic   Prob.
==========================================================================
             C              6.747257     0.152544     44.23158     0.0000
          LOG(MIU)          1.965246     0.047851     41.07000     0.0000
==========================================================================
R-squared                   0.998225   Mean dependent var        2.994654
Adjusted R-squared          0.997633   S.D. dependent var        5.613898
S.E. of regression          0.273139   Akaike info criterion     0.531504
Sum squared resid           0.223815   Schwarz criterion         0.375279
Log likelihood              0.671241   F-statistic               1686.745
Durbin-Watson stat          2.274837   Prob(F-statistic)         0.000032
==========================================================================
```

IV. Calibrating the preference function and discounting

a. General Description

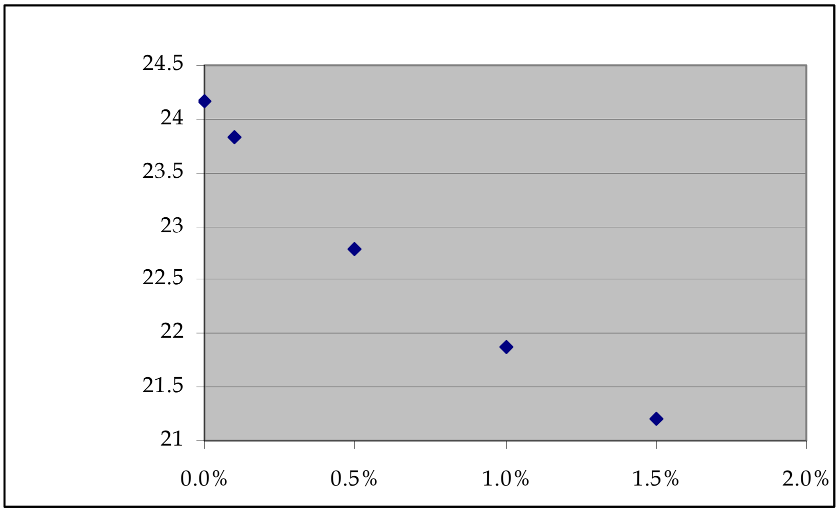
Calibrating the utility function is one of the major new approaches in DICE-2007. The major assumption is that the two taste parameters need to be set in a way that is consistent with observed market returns. This raises deep questions that are discussed in the literature.

b. Lab Notes on the Calibration

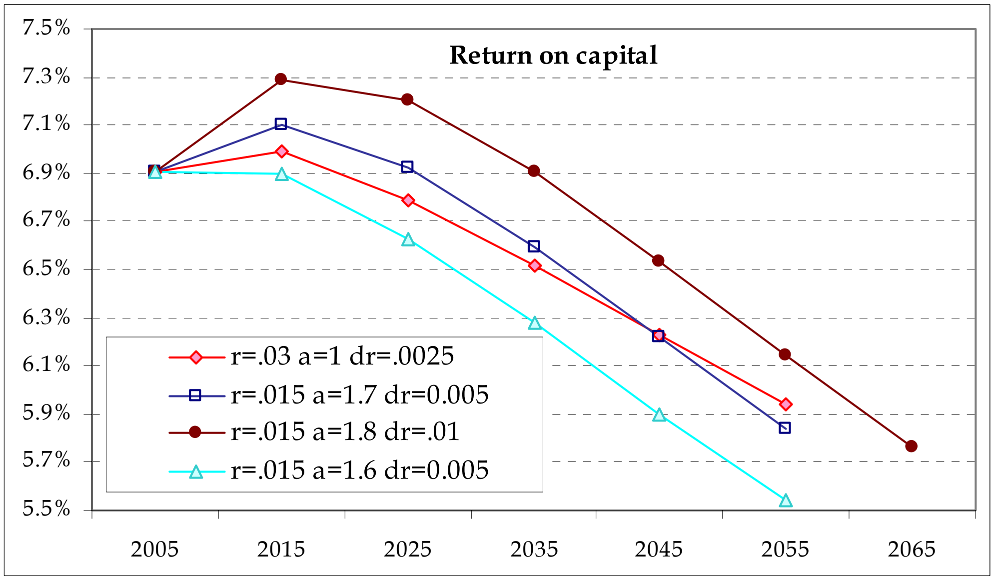
For this calibration, we adjust the elasticity of the MU of consumption to fit the path of interest rates conditional on the time discount rate. We begin with a time path of the real return on capital. The calibration was that the first four period give an average rate of return of around 6.5 percent. The following shows a table with results:

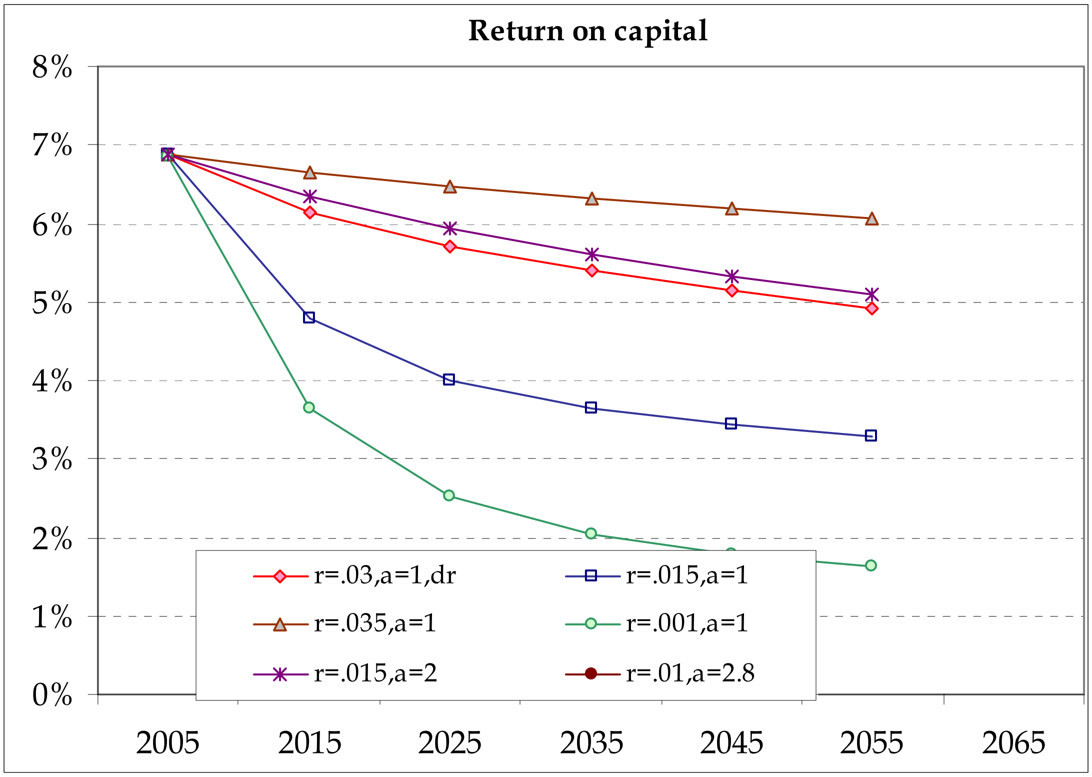
prstp	alpha	r1	r2	r3	r4	av	ctax
0.0%	***2.93***	***6.80%***	***6.60%***	***6.30%***	***6.00%***	***6.43%***	***24.1651***
1.0%	2.50	6.80%	6.80%	6.70%	6.40%	6.68%	
1.0%	2.00	6.80%	6.20%	5.70%	5.40%	6.03%	
1.5%	2.50	6.80%	7.20%	7.20%	7.10%	7.08%	
1.5%	**2.00**	**6.800%**	**6.600%**	**6.300%**	**6.000%**	***6.425%***	**21.2003**
1.0%	2.31	6.800%	6.600%	6.300%	6.000%	6.425%	21.8687
0.5%	2.62	6.800%	6.600%	6.300%	6.000%	6.425%	22.7858
0.1%	2.87	6.800%	6.600%	6.300%	6.000%	6.425%	23.8328
0.0%	2.93	6.800%	6.600%	6.300%	6.000%	6.425%	24.1651

The columns are the time discount rate, consumption elasticity, first four period real returns, average return, and carbon tax. (These were a little different from the final runs). The last five rows show that the initial carbon tax is slightly affected by the calibration, with zero discount rates giving about 15 percent higher initial carbon tax than our base run. There is a slight increase in the current C tax with lower PRSTP with given interest rates, presumably because of higher decline in r in the future. However, to a first approximation, it is the near-term interest rate that determines the current policy, not the discount rate. We can see the impact of the PRSTP on the current carbon tax as follows:



The following shows how real returns differ with different parameters (these have slightly different other parameters, but the pattern is similar to the latest version):




Return on capital
8%
7%
6%
5%
4%
3%
2%
1%
0%
2005
2015
2025
2035
2045
2055
2065
r=.03,a=1,dr
r=.015,a=1
r=.035,a=1
r=.001,a=1
r=.015,a=2
r=.01,a=2.8

Carbon tax	2005	2015	2025	2035	2045	2055
r=.03 a=1 dr=.0025	19.3	29.7	43.6	61.4	83.5	110.4
r=.015 a=1.7 dr=0.005	20.2	31.8	47.6	68.7	95.9	129.9
r=.015 a=1.6 dr=0.005	22.6	35.5	53.0	76.1	105.6	142.1
r=.015 a=1.3 dr=.005	32.7	50.9	74.7	104.9	142.3	187.5
r=.02 a=1.5 dr=.01	23.6	37.8	57.8	85.0	120.2	164.7
r=.015 a=1.8 dr=.01	18.2	28.5	43.0	62.3	87.3	118.9
r=.005 a=2.1	19.6	30.6	45.4	65.0	89.9	120.9
r=.001 a=2.3	19.8	31.0	46.4	67.0	93.4	126.6

Savings rate						
r=.03 a=1 dr=.0025	22.2%	22.0%	21.8%	21.7%	21.7%	21.7%
r=.015 a=1.7 dr=0.005	21.8%	21.7%	21.8%	21.9%	22.0%	22.2%
r=.015 a=1.6 dr=0.005	22.4%	22.3%	22.4%	22.4%	22.6%	22.7%
r=.015 a=1.3 dr=.005	24.6%	24.4%	24.2%	24.2%	24.3%	24.4%
r=.02 a=1.5 dr=.01	21.8%	22.1%	22.3%	22.6%	22.9%	23.2%
r=.015 a=1.8 dr=.01	21.2%	21.2%	21.2%	21.3%	21.5%	21.7%
r=.005 a=2.1	21.9%	21.7%	21.7%	21.7%	21.8%	22.0%
r=.001 a=2.3	21.8%	21.6%	21.6%	21.7%	21.9%	22.1%

Return on capital						
r=.03 a=1 dr=.0025	6.91%	6.99%	6.79%	6.52%	6.23%	5.94%
r=.015 a=1.7 dr=0.005	6.91%	7.10%	6.92%	6.59%	6.22%	5.84%
r=.015 a=1.6 dr=0.005	6.91%	6.90%	6.63%	6.28%	5.90%	5.54%
r=.015 a=1.3 dr=.005	6.91%	6.26%	5.76%	5.35%	4.99%	4.67%
r=.02 a=1.5 dr=.01	6.91%	7.09%	6.81%	6.38%	5.90%	5.44%
r=.015 a=1.8 dr=.01	6.91%	7.29%	7.20%	6.91%	6.53%	6.14%
r=.005 a=2.1	6.91%	7.06%	6.90%	6.60%	6.26%	5.92%
r=.001 a=2.3	6.91%	7.10%	6.95%	6.65%	6.28%	5.91%

Emissions control rate						
r=.03 a=1 dr=.0025	10.2%	13.2%	16.5%	20.2%	24.3%	28.8%
r=.015 a=1.7 dr=0.005	10.5%	13.7%	17.4%	21.6%	26.3%	31.5%
r=.015 a=1.6 dr=0.005	11.2%	14.5%	18.4%	22.8%	27.7%	33.1%
r=.015 a=1.3 dr=.005	13.7%	17.8%	22.3%	27.3%	32.7%	38.7%
r=.02 a=1.5 dr=.01	11.4%	15.1%	19.3%	24.3%	29.8%	36.0%
r=.015 a=1.8 dr=.01	9.9%	12.9%	16.4%	20.4%	25.0%	30.0%
r=.005 a=2.1	10.3%	13.4%	16.9%	20.9%	25.4%	30.3%
r=.001 a=2.3	10.4%	13.5%	17.1%	21.3%	25.9%	31.1%

Discount rate						
r=.03 a=1 dr=.0025		-2.96%	-2.88%	-2.81%	-2.74%	-2.67%
r=.015 a=1.7 dr=0.005		-1.49%	-1.42%	-1.35%	-1.28%	-1.22%
r=.015 a=1.6 dr=0.005		-1.49%	-1.42%	-1.35%	-1.28%	-1.22%
r=.015 a=1.3 dr=.005		-1.49%	-1.42%	-1.35%	-1.28%	-1.22%
r=.02 a=1.5 dr=.01		-1.98%	-1.79%	-1.62%	-1.47%	-1.33%
r=.015 a=1.8 dr=.01		-1.49%	-1.42%	-1.35%	-1.28%	-1.22%
r=.005 a=2.1		-0.50%	-0.50%	-0.50%	-0.50%	-0.50%
r=.001 a=2.3		-0.10%	-0.10%	-0.10%	-0.10%	-0.10%

c. Further notes on analysis of the preference function

Calibration of the utility function is a difficult issue that involves both the discount rate and the elasticity of the marginal utility of consumption. The background from the earlier model is the following. The original DICE model publication had an extensive discussion of discounting issues. In *Managing the Global Commons,* I summarized as follows:

> Beginning with the fundamentals, a *discount rate* is a pure number per unit time that allows us to convert values in the future into values today. The most common form of discount rate is the nominal or money interest rate, which is applied to future dollar values so that they can be converted into present values. When the nominal interest rate is corrected for inflation, we obtain the real interest rate, which represents the rate used to convert future constant-dollar values into today's dollars.
>
> To understand the economics of real interest rates, economists often use the optimal-growth framework that underlies this study, namely, the Ramsey model. The Ramsey model derives the real interest rate, or the *discount rate on goods*, from a combination of time discounting, the elasticity of marginal utility, and growth in consumption. We begin with the fact that society shows different levels of concern about real incomes of different generations. We call this phenomenon *time discounting*. For example, if, as between equally well-off generations, society is indifferent between an increment of real income today and $(1+\rho)^{100}$ increments in 100 years, we say that the "pure rate of time preference" is ρ per year. Most economists and political philosophers find it hard to defend a pure rate of time preference above zero on ethical grounds (just as it may be hard to defend the fact that the United States devotes only 1/20th of one percent of its national income to overseas development assistance). On the other hand, it would be unrealistic to make decisions based on the premise that there is in fact no time preference given that many social decisions are in fact tilted in favor of present generations.
>
> A second and more justifiable reason for favoring present consumption, or in the context of this study postponing GHG control costs, comes from the fact that different generations enjoy different levels of consumption. Industrial countries have witnessed more or less continual growth in living standard for more than a century; thus the per capita real consumption in the U.S. has growth by a factor of four over the twentieth century. Society might well feel that it is appropriate for later, richer generations to pay a larger fraction of GHG control costs, just as high-income people pay a larger fraction of their income in income taxes. We might then discount future costs if average living standards were improving -- a phenomenon we call *growth discounting.*
>
> One must take into account both time and growth discounting to understand the phenomenon of *goods discounting*. Goods discounting refers to the real interest rate defined above and concerns the relative valuation of units of goods or consumption at different points in time; the real interest rate combines goods discounting and time discounting.
> If we follow the approach set out in the Ramsey model developed in Chapter 2, we can derive the condition for an optimal path of investment and consumption as:
>
> $$\partial\{u'[c(t)]\}/\partial t = u'[c(t)] \; \{\partial Y(t)/\partial K(t) - \delta_K - \rho\} \qquad (6.2)$$
>
> which states that the time rate of change of the marginal utility of consumption equals the marginal utility of consumption times the net marginal product of capital $\{\partial Y(t)/\partial K(t) - \delta_K\}$ minus the pure rate of

> social time preference (ρ). Assuming riskless competitive markets, the net marginal product of capital will equal the instantaneous real interest rate [r(t)], so (6.2) reduces to:
>
> (6.3) $r(t) = \partial\{u'[c(t)]\}/\partial t \ / \ u'[c(t)] + \rho = \alpha g(t) + \rho$
>
> where g(t) is the growth rate of per capita consumption. In steady state, with stable population and a constant rate of growth of per capita consumption, (6.3) becomes:
>
> (6.4) $r^* = \alpha g^* + \rho$
>
> where asterisks represent steady-state values. In equation (6.4), if $\rho = .03$, $g = .03$, and $\alpha = 1$ -- parameters which might reflect today's conditions -- the real interest rate on goods would be 6 percent. However, if all economic growth ceases and the growth discount evaporates, the real interest rate would fall to 3 percent. *It is crucial to understand that a high real interest rate can be generated either by a high rate of time preference or by a high elasticity of marginal utility in a society where living standards are improving*….
>
> The methodology of this study is to anchor the parameters in actual observations wherever possible, and evidence on rates of return are reviewed in the next section. We have constructed the DICE model so that the parameters on the right-hand side of equation (6.3) are constrained to be consistent with observed market returns as represented by the left-hand side of (6.3). The base parameters in the model determine a real interest rate of around 6 percent per annum in the first few periods, with the rate declining slowly in the future as population and economic growth slows. In the next section, we discuss the appropriateness of this approach. (p. 122-25)

In other words, the calibration relied primarily on an estimate of the rate of return on capital. Additionally, however, there were difficulties in using an alternative elasticity, as I explained in the review of the Stern Review:

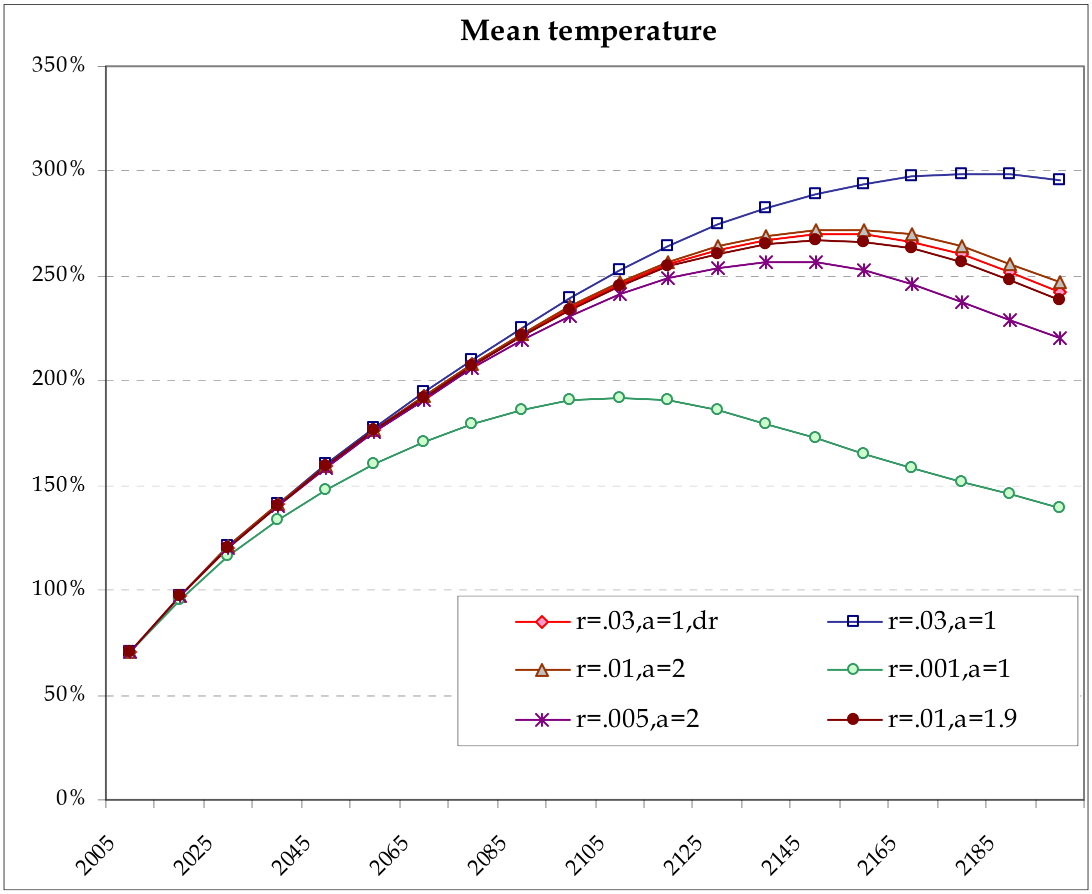
> When the DICE model was constructed fifteen years ago, I assumed logarithmic utility for computational reasons – alternative utility functions would not converge numerically. This calibration led to a social discount rate of 3 percent per year, which was calibrated to match the growth of consumption, savings rates, and market rates of return on capital. Because of improvements in computers and software, we can now easily calibrate alterative utility functions. Experiments with the DICE-2006 model indicate that a social discount rate of 0.1 percent per year is consistent with a utility curvature parameter of 2.25. However, the *Review's* social discount rate of 0.1 percent per year is inconsistent with its utility curvature assumption of 1. The *Review's* calibration gives too low a rate of return and too high a savings rate compared to macroeconomic data, but the alternative calibration proposed here fits the macroeconomic data underlying the DICE model. (Stern Review, pp. 16-17)

We must not make a virtue out of necessity. Sound policy should not rely upon assumptions that are unnecessarily philosophically justifiable. In this round of modeling, I therefore continue to rely on the fundamental *empirical* assumption that the rate of return must be calibrated to observed market levels. However, from the point of view of *welfare economics,* it seems preferable to change the assumption about the pure rate of social time preference to a level that is much closer to zero or generationally neutral. Many paradoxes and puzzles arise with discount rates near zero. We take a pure rate of time preference of 1 percent per year to be a reasonable assumption for the current round. We also compare with time preference rates of 0.5 and 0.1 percent.

Lowering the PRSTP requires, however, that the elasticity of the marginal utility of consumption (EMUC) be changed. For this purpose, I select a EMUC that, along with the assumed PRSTP, yields a rate of return on capital that is stable in the first three periods indicating long-run growth equilibrium. This yields a rate of return of approximately 7 percent per year in the first three decades with the current data. This rate is consistent with safe returns on capital in advanced economies, although it may be lower than estimated returns in high-growth economies such as China (see particularly the rates of return on human capital estimated by Heckman, http://www.journals.uchicago.edu/EDCC/journal/issues/v51n4/510401/510401.html for a very big example).

The following shows the combination of parameters and values of key variables in early periods. There is not much operational difference among the calibrations for rho of 0.1, 0.5 and 1.0 percent per year for the first half century. The first-period carbon taxes are all between $19 and $20 per ton C, which is virtually identical to the base period. So it is mainly a matter of aesthetics. The long-run temperature profiles are shown here. There is some difference in the profiles after a century or so because the savings rates are slightly higher for the low discount rate cases as growth slows.

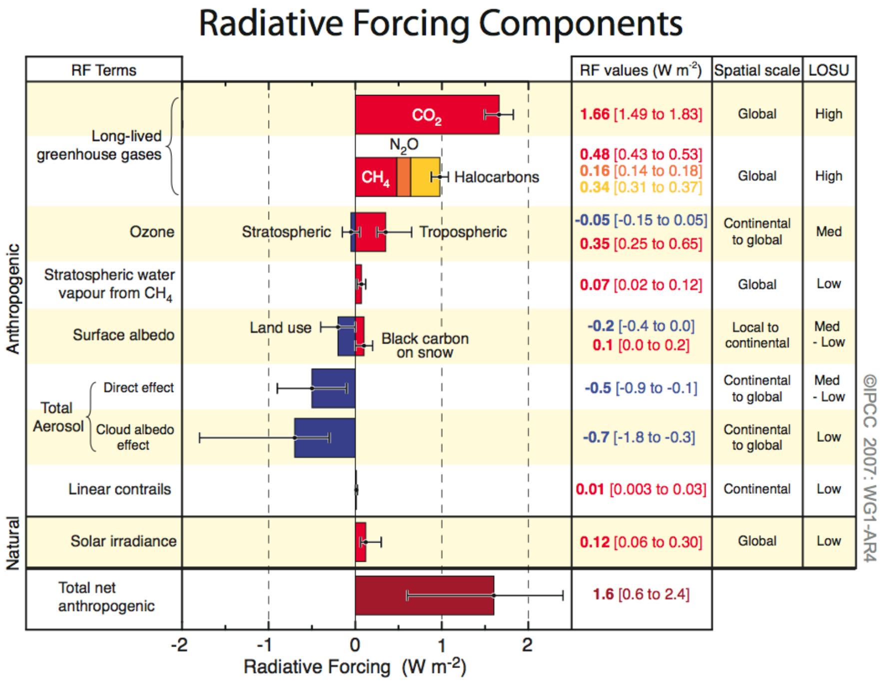
Taking all this information, unless the economic data are revised significantly, the best parameterization appears to be a PRSTP of 1 percent along with a EMUC of 1.85.


Mean temperature
350%
300%
250%
200%
150%
100%
50%
0%
2005
2025
2045
2065
2085
2105
2125
2145
2165
2185
r=.03,a=1,dr
r=.03,a=1
r=.01,a=2
r=.001,a=1
r=.005,a=2
r=.01,a=1.9

Carbon tax	2005	2015	2025	2035	2045	2055
r=.03,a=1,dr	19.3	29.7	43.6	61.4	83.5	110.4
r=.03,a=1	16.9	25.6	36.6	50.3	66.8	86.3
r=.01,a=2	16.8	26.0	38.4	54.8	75.6	101.3
r=.001,a=1	117.1	169.2	225.0	285.5	351.3	422.1
r=.005,a=2	21.8	34.0	50.3	71.7	98.6	131.8
r=.01,a=1.9	19.5	30.1	44.4	62.9	86.2	114.6
r=.005,a=2.1	19.6	30.6	45.4	65.0	89.9	120.9
r=.001,a=2.3	19.8	31.0	46.4	67.0	93.4	126.6
Savings rate						
r=.03,a=1,dr	22.2%	22.0%	21.8%	21.7%	21.7%	21.7%
r=.03,a=1	22.1%	21.7%	21.3%	21.0%	20.8%	20.6%
r=.01,a=2	21.2%	21.0%	20.9%	20.9%	21.0%	21.1%
r=.001,a=1	33.0%	31.7%	31.0%	30.5%	30.2%	30.0%
r=.005,a=2	22.5%	22.3%	22.3%	22.3%	22.4%	22.5%
r=.01,a=1.9	22.1%	21.9%	21.8%	21.7%	21.8%	21.8%
r=.005,a=2.1	21.9%	21.7%	21.7%	21.7%	21.8%	22.0%
r=.001,a=2.3	21.8%	21.6%	21.6%	21.7%	21.9%	22.1%
Return on capital						
r=.03,a=1,dr	6.91%	6.99%	6.79%	6.52%	6.23%	5.94%
r=.03,a=1	6.91%	7.01%	6.91%	6.73%	6.55%	6.37%
r=.01,a=2	6.91%	7.28%	7.24%	7.01%	6.70%	6.38%
r=.001,a=1	6.89%	4.34%	3.36%	2.89%	2.61%	2.41%
r=.005,a=2	6.91%	6.87%	6.63%	6.30%	5.96%	5.62%
r=.01,a=1.9	6.91%	7.00%	6.83%	6.55%	6.23%	5.92%
r=.005,a=2.1	6.91%	7.06%	6.90%	6.60%	6.26%	5.92%
r=.001,a=2.3	6.91%	7.10%	6.95%	6.65%	6.28%	5.91%
Emissions control rate						
r=.03,a=1,dr	10.2%	13.2%	16.5%	20.2%	24.3%	28.8%
r=.03,a=1	9.5%	12.1%	15.0%	18.1%	21.5%	25.1%
r=.01,a=2	9.5%	12.2%	15.4%	19.0%	23.0%	27.5%
r=.001,a=1	27.8%	34.6%	41.1%	47.6%	54.1%	60.6%
r=.005,a=2	10.9%	14.2%	17.9%	22.1%	26.7%	31.8%
r=.01,a=1.9	10.3%	13.3%	16.7%	20.5%	24.8%	29.4%
r=.005,a=2.1	10.3%	13.4%	16.9%	20.9%	25.4%	30.3%
r=.001,a=2.3	10.4%	13.5%	17.1%	21.3%	25.9%	31.1%

V. Lab notes for other GHG forcings

This is clearly an area of major scientific uncertainty. We adopted the other forcings from the FAR:



The total non-CO2 forcings are way down from the TAR, possibly because of cloud albedo effects. We lower the overall level and keep the GISS change in the total over the 2000-2100 period the same. Note that CO2 forcings are more than 10 times other forcings over the period.

VI. Lab notes for the Climate and Carbon-Cycle Module

a. Basic philosophy

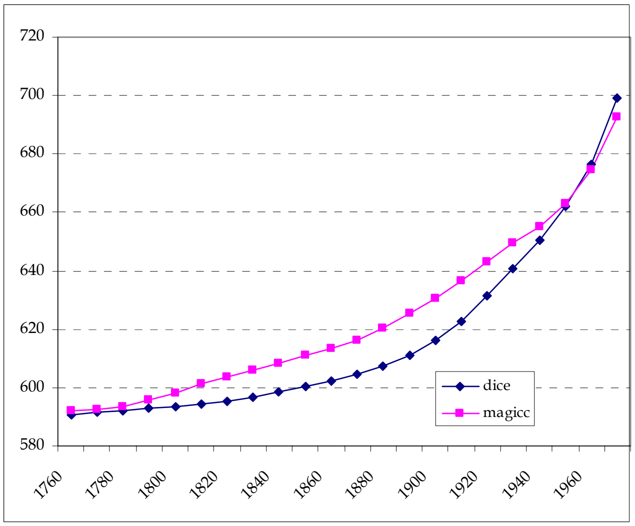
We decided to keep the same basic structure for the climate model, but we used the MAGICC program to tune the parameters. For this purpose, we did the tuning in two stages. First, we set the parameters of the carbon cycle to match the projections for concentrations in the MAGICC model. We did this for a number of runs, but concentrated on the A1F1 run.

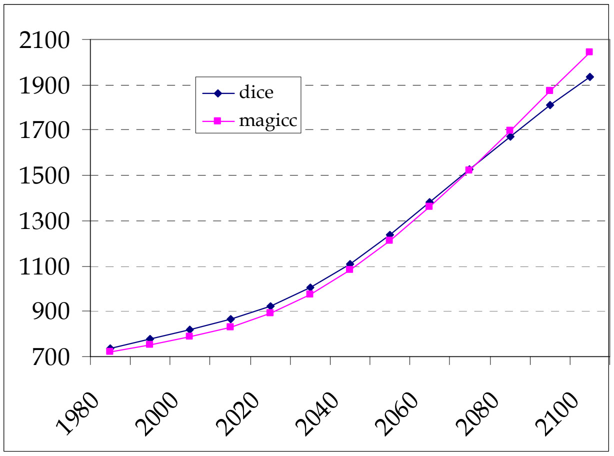
We next took the forcings from the CO2 and exogenous forcings and compared the DICE climate model with the various MAGICC models. We then tuned the DICE parameters to align with the MAGICC model projections. More details are in the next section.

b. MAGICC calibration: Methodology

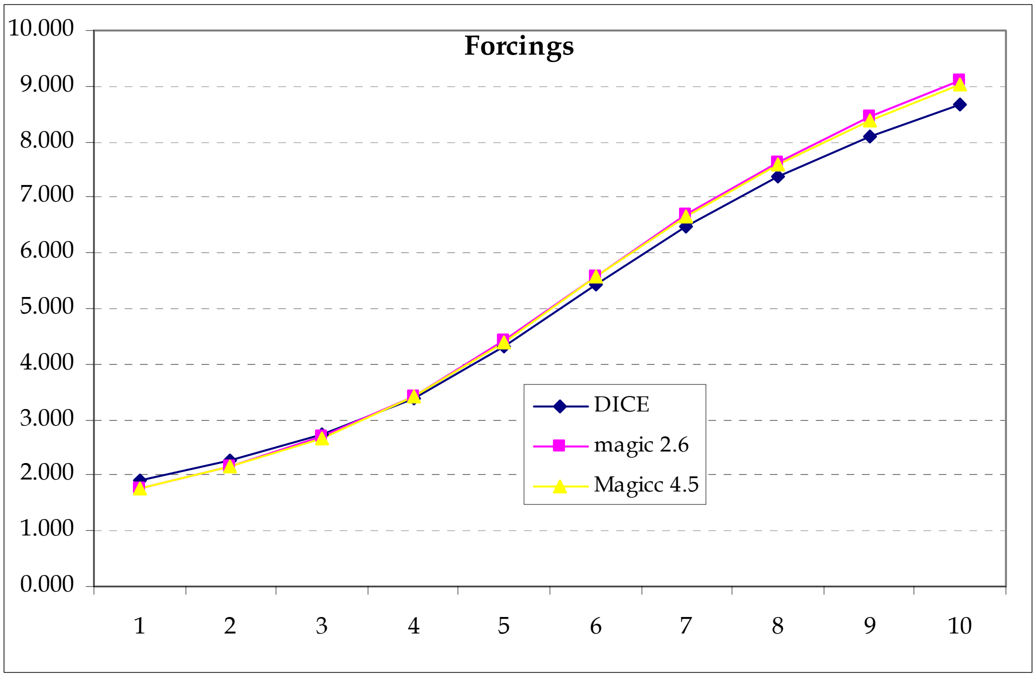
The method of the new recalibration is the following:

1. We based the runs primarily off the A1F1 run in MAGICC. Using this, we looked at the carbon cycle, the forcings, and the temperature predictions for 2.6 and 4.5 degrees for 2x CO2. We could not get the user option with 3 oC to run.
2. We then went step by step through the various steps using a Excel version of the geophysics module. It is somewhat messy, but the files are modelDICE_recalib_v4.4_a1f1.xls and modelDICE_recalib_v4.4_b1min.xls.
3. First, we looked at the carbon cycle from Magicc. We reoptimized the coefficients, assuming that the system was in equilibrium in 1750 (this imposes linearity). This yielded the following comparison for the history and projection. These are imperfect, and particularly the historical is not a good match, perhaps because of our emissions numbers.

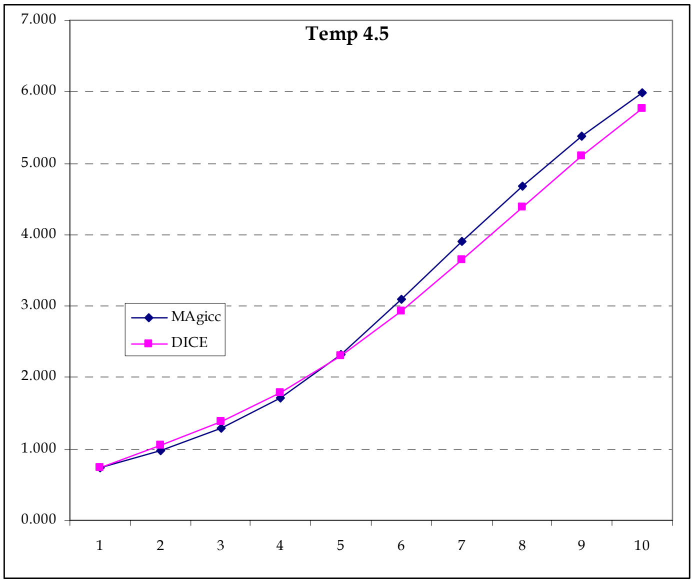

720
700
680
660
640
620
600
580
1760
1780
1800
1820
1840
1860
1880
1900
1920
1940
1960
dice
magicc

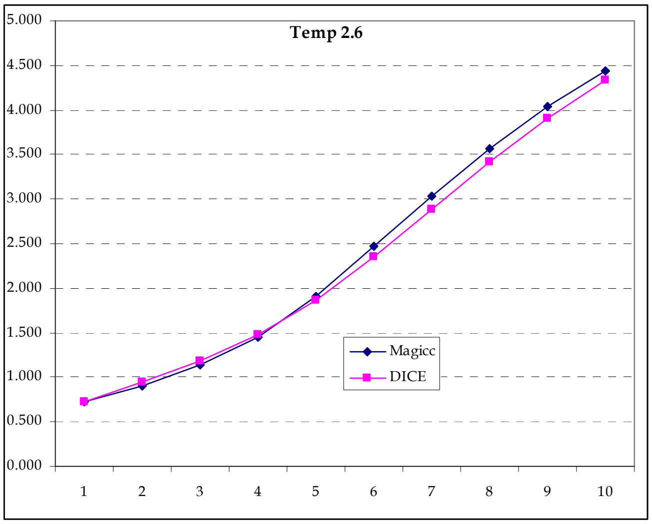

2100
1900
1700
1500
1300
1100
900
700
1980
2000
2020
2040
2060
2080
2100
dice
magicc

4. We then checked the forcings, which were reasonable:

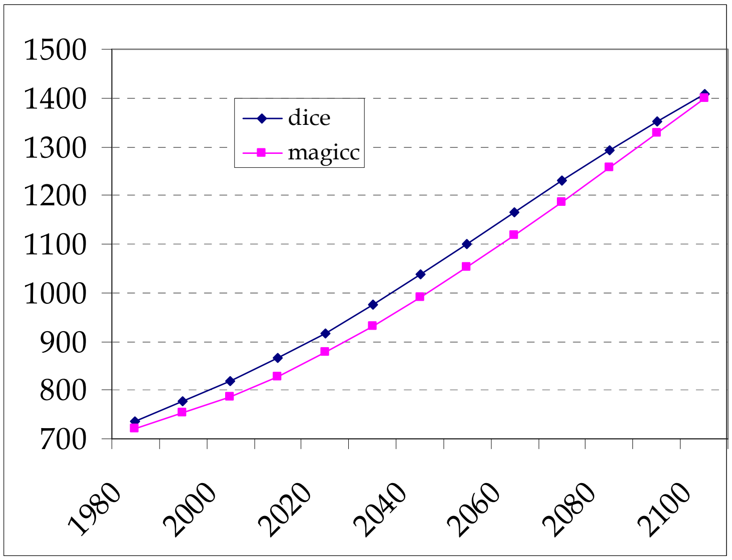


5. We then checked the temperature projections in magicc and dice and recalibrated. This ended up with the following. DICE is a little low, but this reflects the slightly lower forcings just shown. This calibration seems reasonable for now.





6. We also looked at the B1min emissions scenario. These also were a close match for the carbon cycle, as shown below:



VII. Lab notes for Impacts and Damage Function

a. General background

The impacts analysis follows the earlier framework in DICE/RICE-2000. We recalibrated the damage estimates. No major changes in the accounting format were introduced. However, the earlier version was somewhat mechanical. We particularly were concerned that the negative damages for some regions were continued at high T increases, and that the catastrophic was not carefully calibrated.

The changes were as follows: (1) We removed the risk aversion from the catastrophic because we would later introduce risk analysis. (2) We introduced Cline's agricultural studies. These were generally more positive than our earlier estimates, and we took a judgmental average. (3) We recalibrated using the new estimates for GDP and population. (4) We took the output weighted as that is the measurement convention.

The major summary table is the following:

Notes to 2007 revisions:

1. All revisions are in blue.
2. The basic sectoral breakdowns are the same.
3. There have been substantial changes in the regional definitions. These are indicated on page "Regions."
However, we have not carefully redefined regions to match new regions. Problems are primarily low and middle income.
These were poorly estimated in last round, so the estimates are probably about the same quality.
4. All numbers have been updated to use 2005 GDPs in PPP and 2005 US dollars per the data on the spreadsheet macro_011807.xls.
5. The major changes are in the catastrophic estimates. See that sheet for discussion.
6. We have parameterized by assuming that the damage function is to the 2th power. It is then calibrated to 2.5 degrees C.
The estimated elasticity is very close to quadratic, so this is a reasonable specification.
7. Last round had negative damages at 0. This is doubtful given all results and has been constrained so that
new linear term is zero. Specification is therefore simply quadratic.
8. We use the output weights.

The new summary table is shown below (linked to the source).

Region	Agriculture	Other vulnerable mkt	Coastal	Health	market time use	Settlements	Total non-catastrophic [2.5 degree]	[6 degrees]	Catastrophic impact [2.5 degree]	[6 degrees]	TOTAL [2.5 degree]	TOTAL [6 degrees]
US	0.03%	0.00%	0.10%	0.02%	-0.28%	0.10%	-0.03%	1.34%	0.94%	4.00%	0.70%	5.34%
WE/Euro	0.03%	0.00%	0.46%	0.02%	-0.43%	0.25%	0.33%	4.26%	1.09%	4.80%	3.65%	9.06%
OHI	-0.05%	-0.32%	0.09%	0.02%	-0.35%	0.10%	-0.50%	4.19%	1.11%	4.80%	-2.79%	8.99%
Russia	-0.82%	-0.80%	0.05%	0.02%	-0.75%	0.05%	-2.25%	3.63%	1.12%	4.80%	-16.36%	8.43%
EE/FSU	0.03%	0.00%	0.01%	0.02%	-0.36%	0.10%	-0.21%	0.76%	0.94%	4.00%	-0.66%	4.76%
Japan	0.02%	0.00%	0.27%	0.02%	-0.31%	0.25%	0.24%	4.04%	1.07%	4.80%	2.96%	8.84%
China	0.02%	0.32%	0.08%	0.09%	-0.26%	0.05%	0.30%	3.92%	1.04%	4.00%	3.33%	7.92%
India	0.32%	0.29%	0.09%	0.40%	0.30%	0.10%	1.51%	6.94%	1.57%	6.00%	13.29%	12.94%
MidEast	0.35%	0.20%	0.04%	0.23%	0.24%	0.05%	1.12%	4.41%	0.96%	4.00%	9.61%	8.41%
SSA	0.67%	0.32%	0.02%	1.00%	0.25%	0.10%	2.35%	9.55%	1.78%	7.00%	20.00%	16.55%
LA	0.42%	0.28%	0.10%	0.32%	-0.04%	0.10%	1.18%	5.16%	1.30%	5.20%	10.47%	10.36%
OthAsia	0.52%	0.21%	0.09%	0.32%	-0.04%	0.10%	1.20%	5.00%	1.23%	5.00%	10.55%	10.00%
Global: 2105 weights: 2.5 oC												
Output weighted							0.61%	3.51%	1.16%	4.72%	1.77%	8.23%
Population weighted							0.97%	5.60%	1.26%	5.05%	2.24%	10.65%

b. Lab notes

The major issue at this stage is that the database for impact studies continues to be relatively small.

One new finding that informed the revision was the Gecon results, W Nordhaus, *Proc Nat Acad Sci (US)*, Feb. 2006, which indicated that the damage function at the current climate was negative rather than positive.

Calibration is the following:

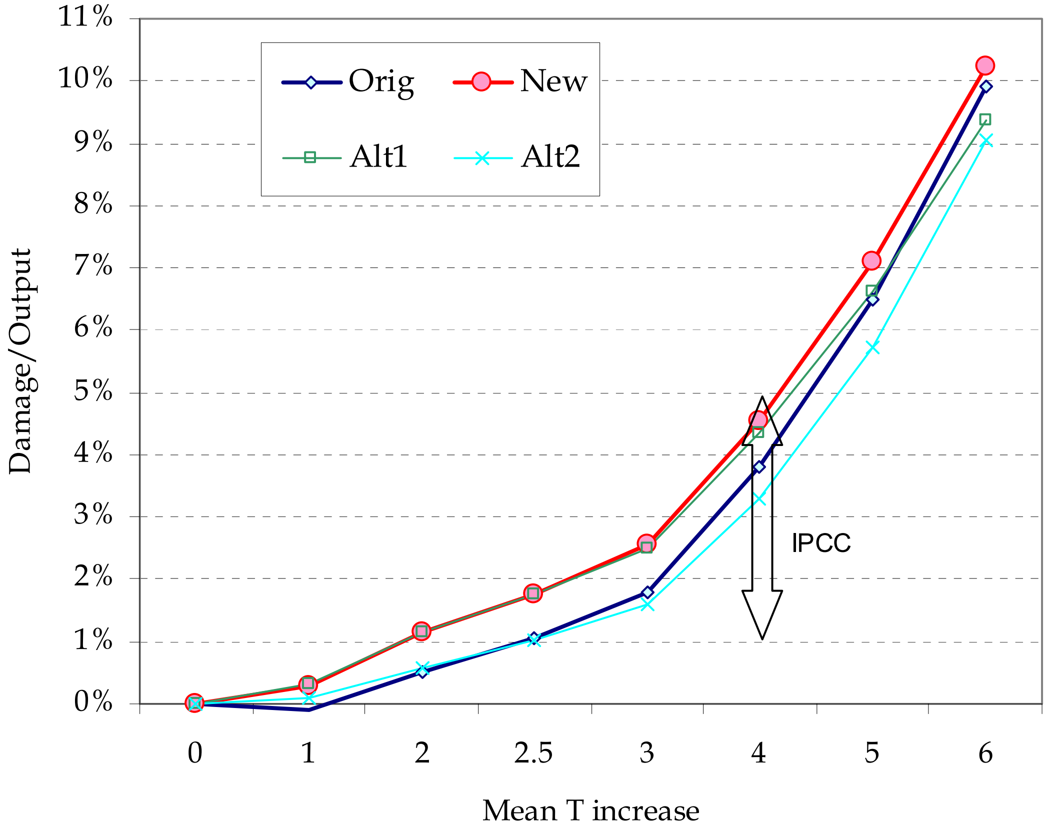
6. To calibrate the new damage function, we set a power function to these estimates. (See below for calibration.) This yields the following:

	Damage estimate			Damage estimate at T*
D/Y=	0.0028388	*T^	2.00000	1.77%
Calibrated at T =	2.5			
D/Y=	0.0010262	*T^	2.50000	9.05%
Calibrated at T =	6			
D/Y=	0.0031112	*T^	1.90000	1.77%
Calibrated at T =	2.5			

Damage functions	Orig	New	Alt1	Alt2	Alt3
Calibrated at	2.5	2.5	2.5	6	
a1	-0.0045	0	0	0	0
a2	0.0035	0.0028388	0.0031112	0.0010262	
a3	2	2	1.9	2.5000000	

T	Orig	New	Alt1	Alt2	Old
0	0.00%	0.00%	0.00%	0.00%	0.00%
1	-0.10%	0.28%	0.31%	0.10%	0.32%
2	0.50%	1.14%	1.16%	0.58%	1.27%
2.5	1.06%	1.77%	1.77%	1.01%	1.98%
3	1.80%	2.55%	2.51%	1.60%	2.85%
4	3.80%	4.54%	4.33%	3.28%	5.07%
5	6.50%	7.10%	6.62%	5.74%	7.93%
6	9.90%	10.22%	9.36%	9.05%	11.41%

Just for information, the following shows the damage functions at a function of T, which suggested IPCC TAR and FAR range:



July 24, 2007. There was a small correction in the tabulation in the earlier version, but this did not affect the results.

VIII. Lab notes for output and sigma estimates

a. General philosophy

The general approach was to start with a database on major countries and then to aggregate up to regions. We then further aggregated to the DICE global total. The current data base is 71 major countries totaling about 96 percent of emissions, output, and slightly less of population. The database and calculations are contained in DATA SET 5.16.2007_WNadj_v5.xls.

b. Lab notes on second round estimates

We have the problem of adjusting our RICE country totals for the world total. We decided to keep the global emissions total and the population total. The question of how to treat the remainder is an issue because the missing countries are low income. We have decided to calibrate to the world totals, as shown below:

	2004	2004	2004	2005	2005	2005
	RICE	Total	ratio	RICE	Total	ratio
Population				5456.27	6312.75	0.864325
GDP PPP				53928.25	57,262.56	0.941771
Emissions	7158.667	7375.654	0.970581	7457.55	7683.594	0.970581
	26,247.97	27,043.57	0.970581			
GDP/capita				9.883717	9.070933	1.089603
co2-gdp				0.138286	0.134182	1.03059

We adjusted China's PPP GDP as it was unrealistically high and distorts the overall numbers. The following shows the calculations. We have rescaled the level to 60 percent of the IMF PPP numbers.

		PPP	MER	US data	US-C trade	Ratio	PPP	MER	Ratio
GDP, 2005		8817.00	2224.00			3.96	**4497.82**	**2224.00**	2.02
Exports G&S	0.368		818.43		762.00				
Exports to US	0.47		*384.66*	244.00	243.00				
GDP-exports		7998.57	1405.57			5.69	4497.82	1405.57	**3.20**
							Ratio	**0.51013**	

In earlier versions of PWT we have made estimates for the PRC based upon quasi-benchmark estimates of individual researchers including Irving Kravis, who, in the early 1980s, made PPP estimates for the year, 1975. Because of its large population, much interest has attached to real product estimates for China. The present range of estimates for the PRC would put it as the 2nd or 3rd largest economy in the world. The wide range of PPP estimates for China and the large size of their difference from the exchange rate suggest that substantial uncertainty is associated with these numbers. China participated in an ESCAP benchmark study for 1993 limited to comparisons between Shanghai and Tokyo and Hong Kong and Guangdong. In the end only the Hong Kong-Guangdong comparison was completed. Currently there is a study of prices in 10 cities within China and discussions have taken place about a possible link to the OECD comparisons for 1996; however that has not yet been realized.

Bold is recalculated.
Key assumption is the 3.2 PPP ratio for non-exports.

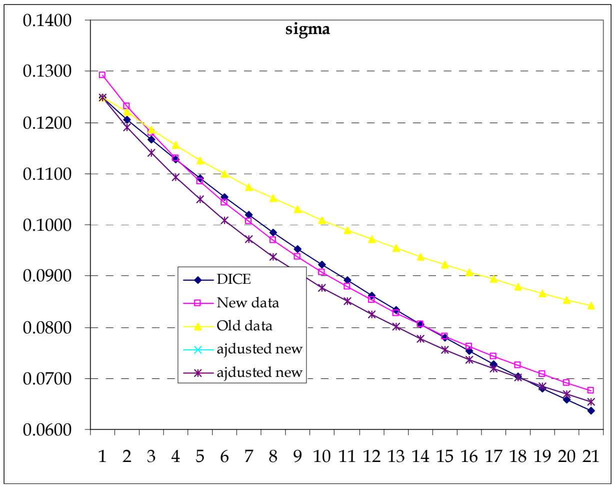
Draft: December, 2001
Alan Heston
Treatment of China in PWT 6.

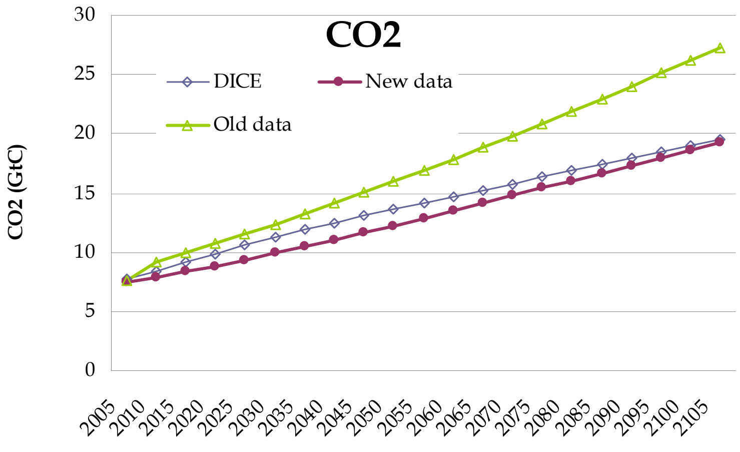
Redid the emissions and output data. Retuned the population to the new IIASA, both by region and by total. The IIASA has more of a hump, which we have not tried to match, but the basic trend is the same.

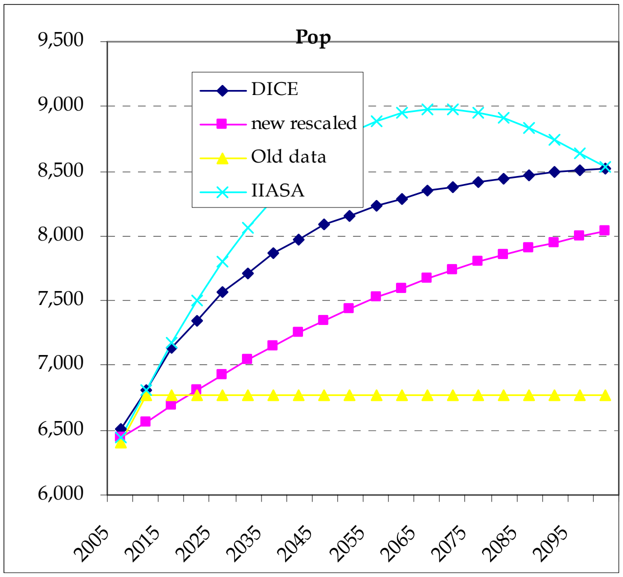
Then we redid the output per capita and emissions per output using the new data. Both have major uncertainties, which cannot be overcome with given data and methods. We have matched reasonably closely the new aggregates (see DATA SET 5.16.2007_WNadj_v3.xls as well as the above file). Output is very slightly lower ($284 tr in 2100 v. $300 in 2100). However, we did overestimate the sigma significantly, and the new 2095 sigma is 0.0681 v. 0.0877. This came about because we did not take into account the long run trend.

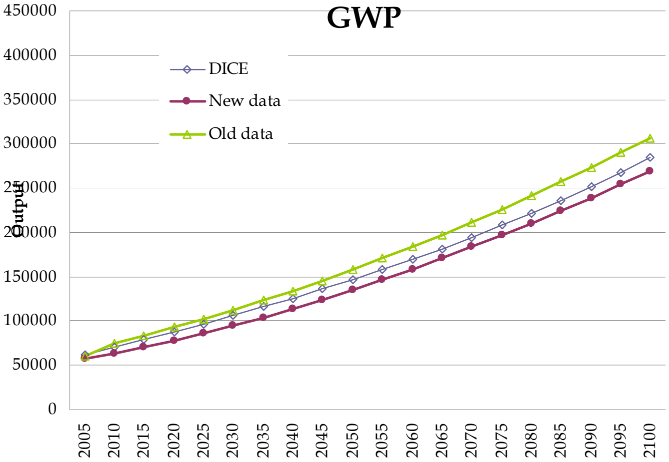
The effect on the social cost of carbon in uncontrolled run is marginally changed, from 29.28 to 30.07. This paradoxical results arises because of consumption elasticity. The runs are pretty stable at this point, even though the trajectories change.

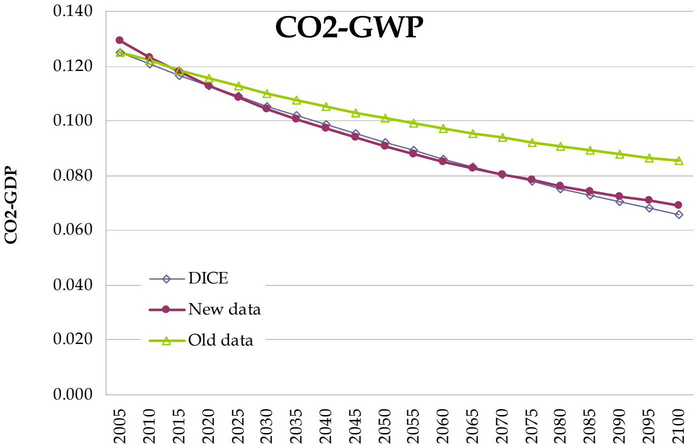
Here is the comparison of old and new data and DICE:





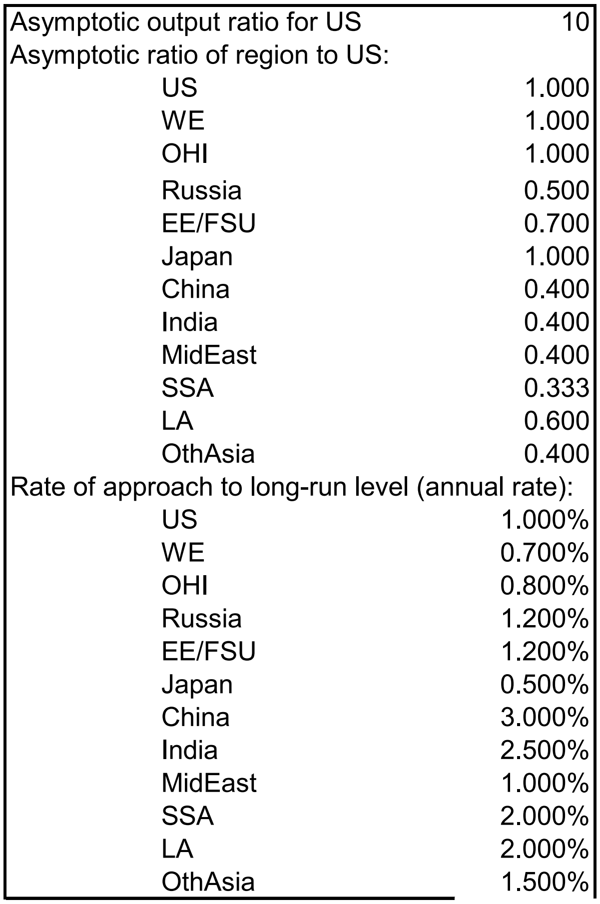

Pop
9,500
9,000
8,500
8,000
7,500
7,000
6,500
6,000
DICE
new rescaled
Old data
IIASA
2005
2015
2025
2035
2045
2055
2065
2075
2085
2095


GWP
Output
450000
400000
350000
300000
250000
200000
150000
100000
50000
0
DICE
New data
Old data
2005
2010
2015
2020
2025
2030
2035
2040
2045
2050
2055
2060
2065
2070
2075
2080
2085
2090
2095
2100


CO2-GWP
CO2-GDP
0.140
0.120
0.100
0.080
0.060
0.040
0.020
0.000
DICE
New data
Old data
2005
2010
2015
2020
2025
2030
2035
2040
2045
2050
2055
2060
2065
2070
2075
2080
2085
2090
2095
2100

c. First round estimates

We have revisited the output and emissions projections. These are contained in macro_011807.xls. The output data are taken from the IMF, World Development Report, although we use PPP aggregation. The TFP growth is adapted from Corderi's estimates. The basic assumptions are the following with respect to the growth of per capita GDP:



Asymptotic output ratio for US		10
Asymptotic ratio of region to US:		
	US	1.000
	WE	1.000
	OHI	1.000
	Russia	0.500
	EE/FSU	0.700
	Japan	1.000
	China	0.400
	India	0.400
	MidEast	0.400
	SSA	0.333
	LA	0.600
	OthAsia	0.400
Rate of approach to long-run level (annual rate):		
	US	1.000%
	WE	0.700%
	OHI	0.800%
	Russia	1.200%
	EE/FSU	1.200%
	Japan	0.500%
	China	3.000%
	India	2.500%
	MidEast	1.000%
	SSA	2.000%
	LA	2.000%
	OthAsia	1.500%

The estimates for emissions are taken from the EIA. We have then projected future emissions growth by region using the following logistics estimates:

Ratio to US	Asymptotic ratio	Ratio asym/initial	Initial growth	
1.00	0.03	0.196	-0.90%	US
1.00	0.03	0.191	-0.90%	WE/Euro
0.25	0.01	0.171	-0.90%	OHI
3.00	0.08	0.256	-1.00%	Russia
1.00	0.03	0.294	-1.00%	EE/FSU
0.50	0.01	0.151	-0.50%	Japan
1.50	0.04	0.312	-0.50%	China
1.30	0.04	0.379	-0.60%	India
3.00	0.08	0.361	-0.30%	MidEast
1.20	0.03	0.176	-0.50%	SSA
1.00	0.03	0.294	-0.50%	LA
1.00	0.03	0.158	-0.50%	OthAsia

We then ran the model out to 2200 using the Hotelling estimates. We calibrated the sigmas and output levels of the new DICE model with the estimates from the aggregated regional RICE. This produced the following calibration, shown in sheet "calibrateDICE2005" of the above Excel sheet:

	DICE MODEL			RICE regional aggregated			Ratio RICE to DICE		
year	2005	2105	2205	2005	2105	2205			
output	61.063	291.445	587.23	61.036	246.571	496.974	1.000	0.846	0.846
pccon	7.432	27.531							
savrate	0.22	0.22							
indem	7.576	19.998	22.49743	7.619	20.282	26.709	1.006	1.014	1.187
sigma	0.125	0.072	0.0456	0.125	0.082	0.072	1.000	1.146	1.583

The emissions are reasonably close, with the sigma and output errors offsetting. Note that the Hotelling rents are important by 2200, so those estimates will not match.

IX. Lab notes on population

a. Background

We got the new population estimates from Wolfgang Lutz. The totals are not all that different, but the distribution is very different from the earlier. We tuned the totals and the uncertainty estimates to the new IIASA projections. These are a little higher than our earlier. The main problem is the shape over the 21st century (see above).

b. Lab notes

The projections are in pop_compare.xls

*****

X. Lab notes specification of participation function

We next add a model for participation. The modeling equations are the following:

```
** Participation
 PARTFRACTper1  Fraction of emissions under control regime 2005 /.1/
 PARTFRACTper21 Fraction of emissions under control regime 2205 /.5/
PARTFRACT(T) = PARTFRACTper1+ .05*(PARTFRACTper21-PARTFRACTper1)*(ORD(T)-1)$(ORD(T) LT
22)+ (PARTFRACTper21-PARTFRACTper1)$(ORD(T) GE 22);
ABATEEQ(T)..    ABATECOST(T) =E=     (PARTFRACT(T)**(1-
expcost2))*YGROSS(T)*(cost1(t)*(MIU(T)**EXPcost2));
*ABATEEQ(T)..   ABATECOST(T) =E=   YGROSS(T)*(cost1(t)*(MIU(T)**EXPcost2));
YY(T)..         Y(T) =E= YGROSS(T)*((1-(PARTFRACT(T)**(1-
expcost2))*cost1(t)*(MIU(T)**EXPcost2)))/(1+a1*TATM(T)+ a2*TATM(T)**2);
```

The general theory is the following: We assume that only a fraction of countries participates in the climate protocols. In this example, the fraction begins at 1 percent and rises to 50 percent in equal increments.

The impact is the following: Assume that the parameter π represents the participation rate. With this participation, if the control fraction of the participants in miu = 1, then the overall miu = π. The cost is the cost of that fraction, given by

C = a*miu^b

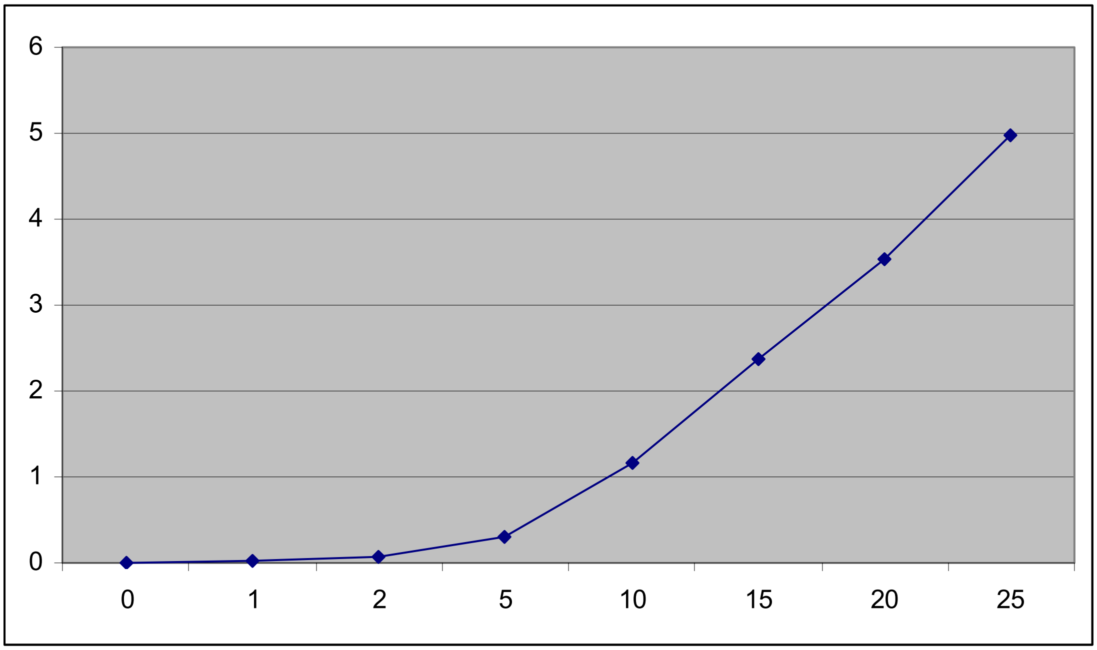
And the total cost is then the following fraction of total reductions:

C(miu) = π a (miu/ π)^b = π^(1-b) a*miu^b

So there is an inefficiency given by the parameter π^(1-b). Because our estimate of b is relatively large (2.8), this means that nonparticipation is relatively costly.

XI. Lab notes for calibration of length of no-control period

Because including carbon constraints is a problem, another approach is to have miu=0 for a limited period, after which the carbon constraints kick in. This is not as efficient as Hotellling pricing, but will get most of the impact. The following shows the impact, with objective function and periods of zero miu. This is very close to the solution where have Hoteling miu for 20 periods and then optimal (for that).



Case	Obj function	Diff from opt (trill)	Diff from opt (bill)
opt	1332.656	0	-
1 period	1332.626	0.03	30
2 periods	1332.578	0.078	78
5 periods	1332.363	0.293	293
10 periods	1331.498	1.158	1,158
15 periods	1330.295	2.361	2,361
20 periods	1329.12	3.536	3,536
	1327.677	4.979	4,979
Optimal after 20 perio	1329.167	3.489	3,489

XII. Lab notes for Importance Of Backstop Price Revision

One of the major differences in the new DICE is the estimate of the backstop technology. This was not a central issue in the earlier DICE model as it was not a variable which affected the optimal policy (which is the central design strategy of the modeling).

In the DICE version of the last round, the backstop (miu=1) rose rapidly over time. The following shows a comparison of estimates of the backstop in the current and the old version (these are approximate):

	MC carbon reductio	
miu=.5	Old	New
2050	$ 323	$ 300
2100	$ 1,100	$ 275
2200	$ 1,480	$ 230
miu=1		
2050	$ 730	$ 915
2100	$ 1,250	$ 833
2200	$ 4,000	$ 740

The current version is much more optimistic about future carbon technologies. It reflects both a modeling error in the last round and much technological work on the issue.

To test the importance of the backstop issue, we ran the new model with a rising rather than falling backstop price. The alternative model was D_011207d_opt.GMS, while the new model was D_011207c_opt.GMS. Some results are shown in test_base and opt_011207.xls.

The following shows a rough simulation where we use a rising backstop in the spirit of DICE-1998. The rising backstop means that carbon policies are much more expensive, so that constraining policies are more advantageous in the present circumstance.

OLD

Run	Objective functio	Difference from base/150:	Objective functio
Base			
150 year delay	1327.345	-2.465	0
50 year delay	1,329.28	-3.757	1.932
Optimal	1330.16	-3.893	2.815
Limit to 2 degree C	1322.167	-9.877	-5.178
Limit to 2.5 degree C	1327.451	-6.521	0.106
Limit to 2X CO2	1329.504	-4.549	2.159

NEW

Run	Objective function	Difference from base/150: Objective function
Base		
150 year delay	1329.810	0.000
50 year delay	1333.034	3.224
Optimal	1334.053	4.243
Limit to 2 degree C	1332.044	2.234
Limit to 2.5 degree C	1333.972	4.162
Limit to 2X CO2	1334.053	4.243

Note that the initial carbon taxes are not as heavily affected, however, indicating that the long-term outlook is different but current optimal policy is relatively little affected:

NEW

Carbon tax comparison	2005.00	2055.00	2105.00	2205.00
Base				
150 year delay	0.03	0.36	2.73	407.92
50 year delay				
Optimal	19.18	112.67	329.53	742.31
Limit to 2 degree C	28.75	271.68	677.79	745.95
Limit to 2.5 degree C	19.88	124.45	449.92	743.52
Limit to 2X CO2	19.18	112.67	329.53	742.31
Hotelling	0.03	0.36	2.73	64.53

OLD

Carbon tax comparison	2005.00	2055.00	2105.00	2205.00
Base				
150 year delay	0.00	0.00	0.00	0.00
50 year delay				
Optimal	19.79	120.2	380.2	1893.69
Limit to 2 degree C	36.94	405.81	1,578.71	3,779.72
Limit to 2.5 degree C	23.65	185.47	1048.94	3483.8
Limit to 2X CO2	20.32	130.6	506.4	2949.67

The new model has slightly lower C taxes in the short run (because the long run is more optimistic). There is little short-run difference, but an enormous long-run difference in the optimal policy because emissions reductions in the long-run are economical. The long-run are unrealistically high and would only be applicable in a world where technology is worse that stagnant.

Note primarily that policy is now much more important because the long-run cost of reductions are much less expensive.

XIII. Lab notes on the carbon cycle.

We have recalibrated the carbon cycle to recent data on sources and sinks. We have used GISS data on atmospheric carbon concentrations. We assume that 110 GtC of deforestation has occurred from 1850 to 2005, based on Marland. (IPCC SAR WGII Tables B.3 and B.4 in Sect B.3.3.1 and Gregg Marland, Oak Ridge National Laboratory.) The earlier carbon cycle model overpredicts atmospheric concentrations by a substantial margin given the revised data. The time pattern of land-use emissions is from Joos et al.

We then optimized the coefficients to minimize the squared errors over the last 20 years. The change for the 10-year model is the following for a baseline run with unlimited carbon resources.. The climate trajectory is lower (because the old model overpredicted atmospheric concentrations). The marginal retention rate is in line with other models. The impact on the short-term carbon tax is lower by 4 percent.

Coefficient	Old	New
b11	0.66616	*0.6608035*
b12	0.33384	0.3391965
b21	0.27607	0.2764262
b22	0.60897	*0.5968861*
b23	0.11496	0.1266877
b32	0.00422	0.0038996
b33	0.99578	*0.9961004*

NEW			
year	2005	2105	2205
temp	0.71	2.76	4.65
conc	787.0	1,417.6	2,358.9
ctax	16.9	289.9	1,019.0
indem	75.2	221.9	349.8
Cumulative emission	75.2	1,630.6	4,577.5
Average atmos ret ratio		38.7%	34.3%

OLD	2005	2105	2205
temp	0.71	2.88	4.824
conc	787.00	1,481.30	2,485.61
ctax	17.6	292.4	1,007.5
indem	75.2	221.8	349.2
Cumulative emission	75.2	1,630.3	4,573.4
Average atmos ret ratio		42.6%	37.1%

Change (%)	2005	2105	2205
temp	0.00%	4.42%	3.65%
conc	0.00%	4.49%	5.37%
ctax	3.96%	0.88%	-1.13%
indem	0.00%	-0.05%	-0.18%
Cumulative emission	0.00%	-0.02%	-0.09%
Average atmos ret ratio		10.12%	8.16%

XIV. Lab notes on adjustment of climate module

The following are the lab notes on the adjustment of the climate module:

The basic structure of the climate model has not been changed since the last round. However, there has been much careful analysis of GCMs, particularly in the IPCC TAR, and we have incorporated that in the latest model.

One first change is to incorporate the latest view on forcings. For this, we took the total forcings from GISS. We then examined the model using both GISS and Hadley global temperature record.

We first looked at the average temperature (using the average of GISS and Hadley). We have estimated the model using both GISS and Hadley data. Using GISS total forcings, we get the following results. Constraining the adjustment parameter to .15, the key parameter is c(1), which is t2xco2. This is lower than the estimate from GCMs or our parameterization, which is 2.9. The parameter reasonably well determined in this tight specification. Estimates using well-mixed GHGs are not significant.

The continuing puzzle is why the time series estimate of the temperature-sensitivity coefficient is lower than the models. The estimate is not significantly below 3. Time will tell.

```
Dependent Variable: TAV
Method: Least Squares
Date: 01/11/07   Time: 19:50
Sample (adjusted): 1891 2006
Included observations: 116 after adjustments
Convergence achieved after 4 iterations
TAV=TAV(-10)+.15*( TOTFORC_GISS-(4.1/C(1))*TAV(-10)-.44
        *(TAV(-10)-TLOW(-10))  )
==========================================================
                 Coefficient   Std. Error   t-Statistic   Prob.
==========================================================
        C(1)        2.268675     0.445698      5.090166   0.0000
==========================================================
```

```
R-squared                  0.712229   Mean dependent var      -0.064793
Adjusted R-squared         0.712229   S.D. dependent var       0.243178
S.E. of regression         0.130451   Akaike info criterion   -1.227048
Sum squared resid          1.957022   Schwarz criterion       -1.203310
Log likelihood             72.16878   Durbin-Watson stat       0.906687
======================================================================
```

We also estimate with t2xco2=3 degrees, yielding:

```
Dependent Variable: TAV
Method: Least Squares
Date: 01/11/07   Time: 19:47
Sample (adjusted): 1891 2006
Included observations: 116 after adjustments
TAV=TAV(-10)+C(2)*( TOTFORC_GISS-1.366*TAV(-10)-.44*(TAV(
        -10)-TLOW(-10))  )
======================================================================
                        Coefficient    Std. Error    t-Statistic    Prob.
======================================================================
          C(2)             0.155163      0.017754       8.739675   0.0000
======================================================================
R-squared                  0.708578   Mean dependent var      -0.064793
Adjusted R-squared         0.708578   S.D. dependent var       0.243178
S.E. of regression         0.131276   Akaike info criterion   -1.214443
Sum squared resid          1.981846   Schwarz criterion       -1.190705
Log likelihood             71.43771   Durbin-Watson stat       0.943899
======================================================================
```

Freeing both coefficients yields:

```
Dependent Variable: TAV
Method: Least Squares
Date: 01/11/07   Time: 19:55
Sample (adjusted): 1891 2006
Included observations: 116 after adjustments
Convergence achieved after 4 iterations
TAV=TAV(-10)+C(2)*( TOTFORC_GISS-(4.1/C(1))*TAV(-10)-.44
```

```
        *(TAV(-10)-TLOW(-10))  )
==========================================================================
                          Coefficient    Std. Error    t-Statistic    Prob.
==========================================================================
            C(2)             0.150269      0.018181       8.265170   0.0000
            C(1)             2.271184      0.478918       4.742319   0.0000
==========================================================================
R-squared                    0.712229   Mean dependent var         -0.064793
Adjusted R-squared           0.709705   S.D. dependent var          0.243178
S.E. of regression           0.131022   Akaike info criterion      -1.209808
Sum squared resid            1.957018   Schwarz criterion          -1.162333
Log likelihood               72.16889   Durbin-Watson stat          0.906993
==========================================================================
```

ESTIMATING ALL THREE COEFFICIENTS

```
Dependent Variable: TAV
Method: Least Squares
Date: 01/11/07   Time: 19:57
Sample (adjusted): 1891 2006
Included observations: 116 after adjustments
Convergence achieved after 4 iterations
TAV=TAV(-10)+C(2)*( TOTFORC_GISS-(4.1/C(1))*TAV(-10)-C(3)
        *(TAV(-10)-TLOW(-10))  )
==========================================================================
                          Coefficient    Std. Error    t-Statistic    Prob.
==========================================================================
            C(2)             0.172415      0.022721       7.588394   0.0000
            C(1)             2.721348      0.647524       4.202697   0.0001
            C(3)             1.037278      0.330157       3.141768   0.0021
==========================================================================
R-squared                    0.718649   Mean dependent var         -0.064793
Adjusted R-squared           0.713670   S.D. dependent var          0.243178
S.E. of regression           0.130124   Akaike info criterion      -1.215130
Sum squared resid            1.913357   Schwarz criterion          -1.143916
Log likelihood               73.47752   Durbin-Watson stat          0.913028
==========================================================================
```

ONE YEAR ADJUSTMENT:

Using one-year adjustment, we get the following, with even lower temp sensitivity:

```
Dependent Variable: TAV
Method: Least Squares
Date: 01/11/07   Time: 20:06
Sample (adjusted): 1882 2006
Included observations: 125 after adjustments
Convergence achieved after 6 iterations
TAV=TAV(-1)+C(2)*( TOTFORC_GISS-(4.1/C(1))*TAV(-1)-.044
        *(TAV(-1)-TLOW(-1))  )
==============================================================
                        Coefficient   Std. Error   t-Statistic   Prob.
==============================================================
          C(2)            0.050109     0.013030     3.845539    0.0002
          C(1)            1.239215     0.314128     3.944936    0.0001
==============================================================
R-squared                 0.824945   Mean dependent var     -0.079852
Adjusted R-squared        0.823522   S.D. dependent var      0.241038
S.E. of regression        0.101258   Akaike info criterion  -1.726415
Sum squared resid         1.261148   Schwarz criterion      -1.681162
Log likelihood            109.9009   Durbin-Watson stat      2.356257
==============================================================
```

```
Dependent Variable: TAV-.07*.044*TLOW(-1)
Method: Least Squares
Date: 01/11/07   Time: 20:25
Sample (adjusted): 1882 2006
Included observations: 125 after adjustments
======================================================================
            Variable                Coefficient   Std. Error   t-Statistic   Prob.
======================================================================
  (1-.07*1.366-.07*0.044)*TAV(-1)     0.923321     0.046935     19.67229    0.0000
        .07*TOTFORC_GISS              0.711085     0.184331     3.857664    0.0002
======================================================================
R-squared                             0.825068   Mean dependent var     -0.078965
```

Adjusted R-squared	0.823645	S.D. dependent var	0.241000
S.E. of regression	0.101207	Akaike info criterion	-1.727423
Sum squared resid	1.259877	Schwarz criterion	-1.682170
Log likelihood	109.9639	Durbin-Watson stat	2.359090

There are nasty AR and MA problems in the regressions however. For example:

```
Dependent Variable: TAV-.07*.044*TLOW(-1)
Method: Least Squares
Date: 01/11/07   Time: 20:27
Sample (adjusted): 1882 2006
Included observations: 125 after adjustments
Convergence achieved after 13 iterations
Backcast: 1881
==============================================================================
                Variable                 Coefficient   Std. Error   t-Statistic    Prob.
==============================================================================
   (1-.07*1.366-.07*0.044)*TAV(-1)          1.061581     0.018635      56.96689   0.0000
          .07*TOTFORC_GISS                  0.365049     0.077425      4.714897   0.0000
                MA(1)                      -0.665235     0.073908     -9.000895   0.0000
==============================================================================
R-squared                                   0.848090   Mean dependent var      -0.078965
Adjusted R-squared                          0.845600   S.D. dependent var       0.241000
S.E. of regression                          0.094698   Akaike info criterion   -1.852535
Sum squared resid                           1.094066   Schwarz criterion       -1.784656
Log likelihood                              118.7835   Durbin-Watson stat       1.684536
==============================================================================
Inverted MA Roots                            .67
==============================================================================
```

These might be introduced by the data smoothing, but it is not clear. This makes it difficult to rely upon the regression analysis for parameterization. (I'm not sure if this has turned up in the statistical analysis.)

We tried an alternative specifcation and adjusted it only to 0.15 as a compromise between models and data. The "NEWNEW" is with the adjusted and the "NEW" is with the carbon cycle adjustment only.

NEW

year	2005	**2105**	2205
temp	0.71	**2.758**	4.654
conc	787	**1417.594**	2358.884
ctax	16.9	**289.86**	1019.02
indem	75.1822	**221.9427**	349.8124
Cum emis	75.1822	**1630.577**	4577.455
average ret ratio		**0.386731**	0.343397

NEWNEW

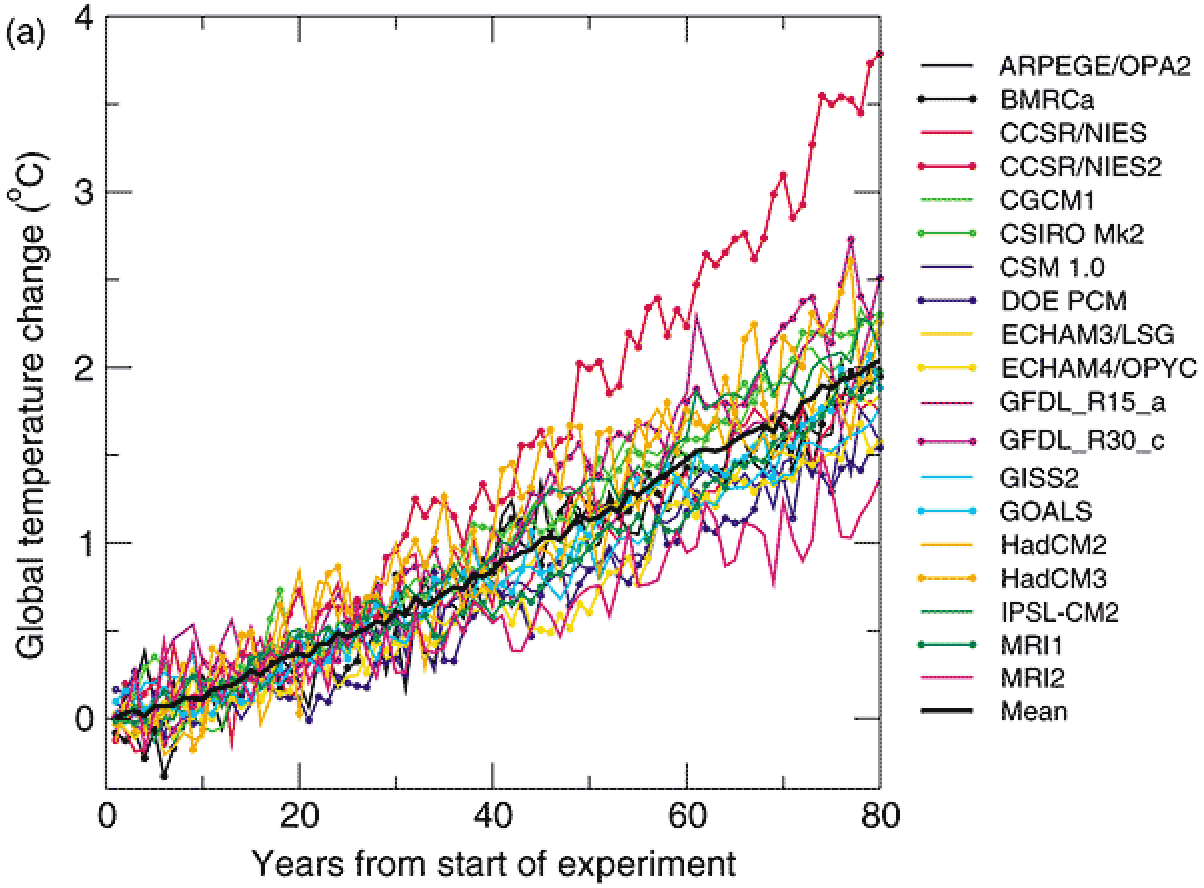
temp	0.71	**2.616**	4.592
conc	787	**1481.758**	2488.904
ctax	13.49	**239.69**	871.04
indem	75.1822	**222.0999**	350.0969

temp	1	**0.948513**	0.986678
conc	1	**1.045263**	1.055119
ctax	0.798225	**0.826916**	0.854782
indem	1	**1.000708**	1.000813

As additional runs, we tested the model against the IPCC TAR model simulations. Using the GISS forcings, we estimate the following patterns of model temperature and forcings:

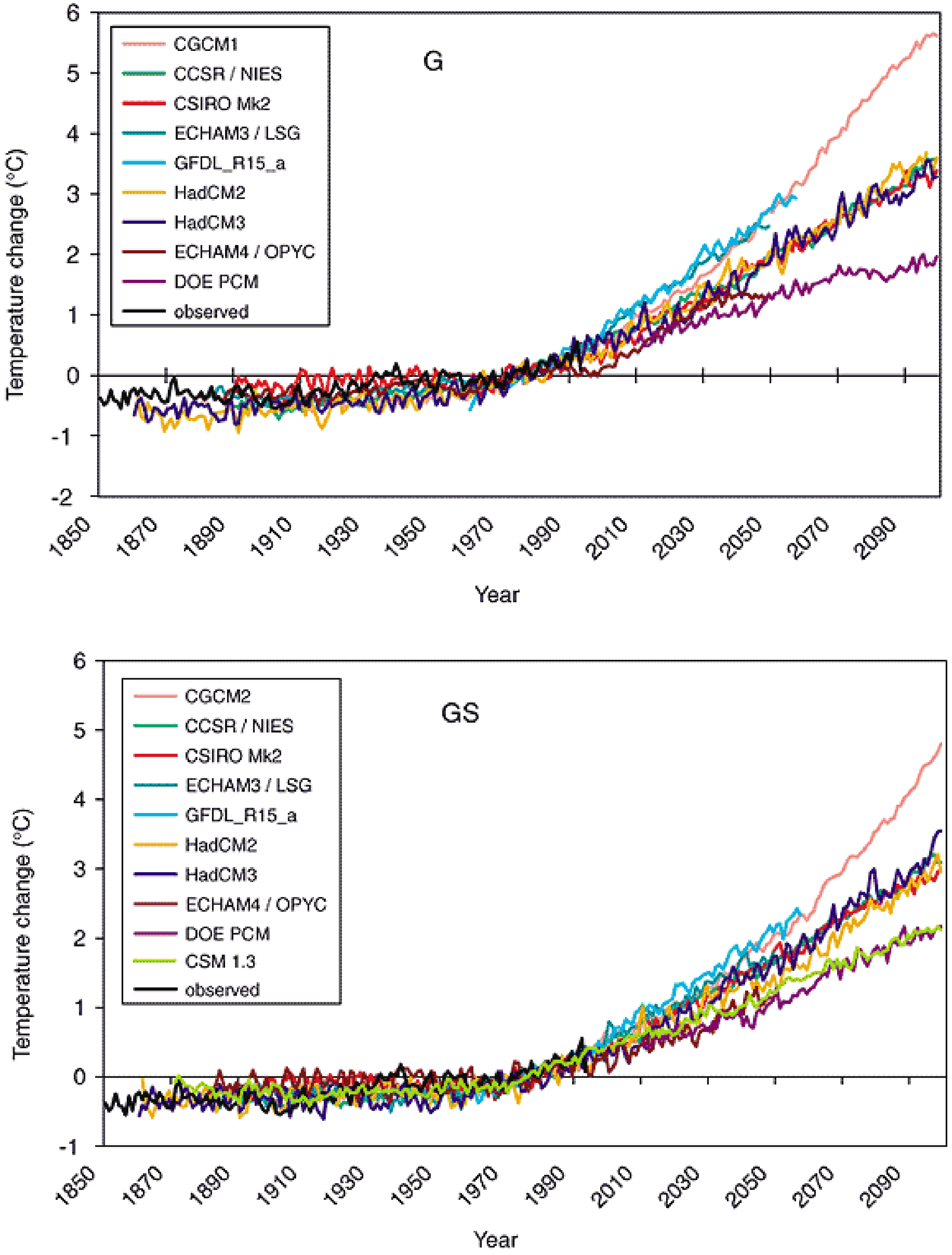
	1905	1915	1925	1935	1945	1955	1965	1975	1985	1995	2005	2000
Atmospheric temperature (degrees Celsius above preindustrial)	0.000	0.010	0.042	0.087	0.143	0.191	0.247	0.337	0.485	0.693	0.920	0.806
Total increase in radiative forcing since preindustrial (Watts per square meter)	0.042	0.160	0.278	0.408	0.476	0.603	0.855	1.277	1.816	2.289	2.539	2.414
Lower ocean temperature (degrees Celsius above preindustrial)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

This matches very closely Figure 8.15 in TAR.



In addition, the 1% co2 growth scenarios matches the TAR, but it has a one-period lag. That is, it gets to the 1.8 degree one period late because of the integration period.

(a)


G
CGCM1
CCSR / NIES
CSIRO Mk2
ECHAM3 / LSG
GFDL_R15_a
HadCM2
HadCM3
ECHAM4 / OPYC
DOE PCM
observed
Temperature change (°C)
6
5
4
3
2
1
0
-1
-2
1850
1870
1890
1910
1930
1950
1970
1990
2010
2030
2050
2070
2090
Year
GS
CGCM2
CCSR / NIES
CSIRO Mk2
ECHAM3 / LSG
GFDL_R15_a
HadCM2
HadCM3
ECHAM4 / OPYC
DOE PCM
CSM 1.3
observed
Temperature change (°C)
6
5
4
3
2
1
0
-1
1850
1870
1890
1910
1930
1950
1970
1990
2010
2030
2050
2070
2090
Year

(b)

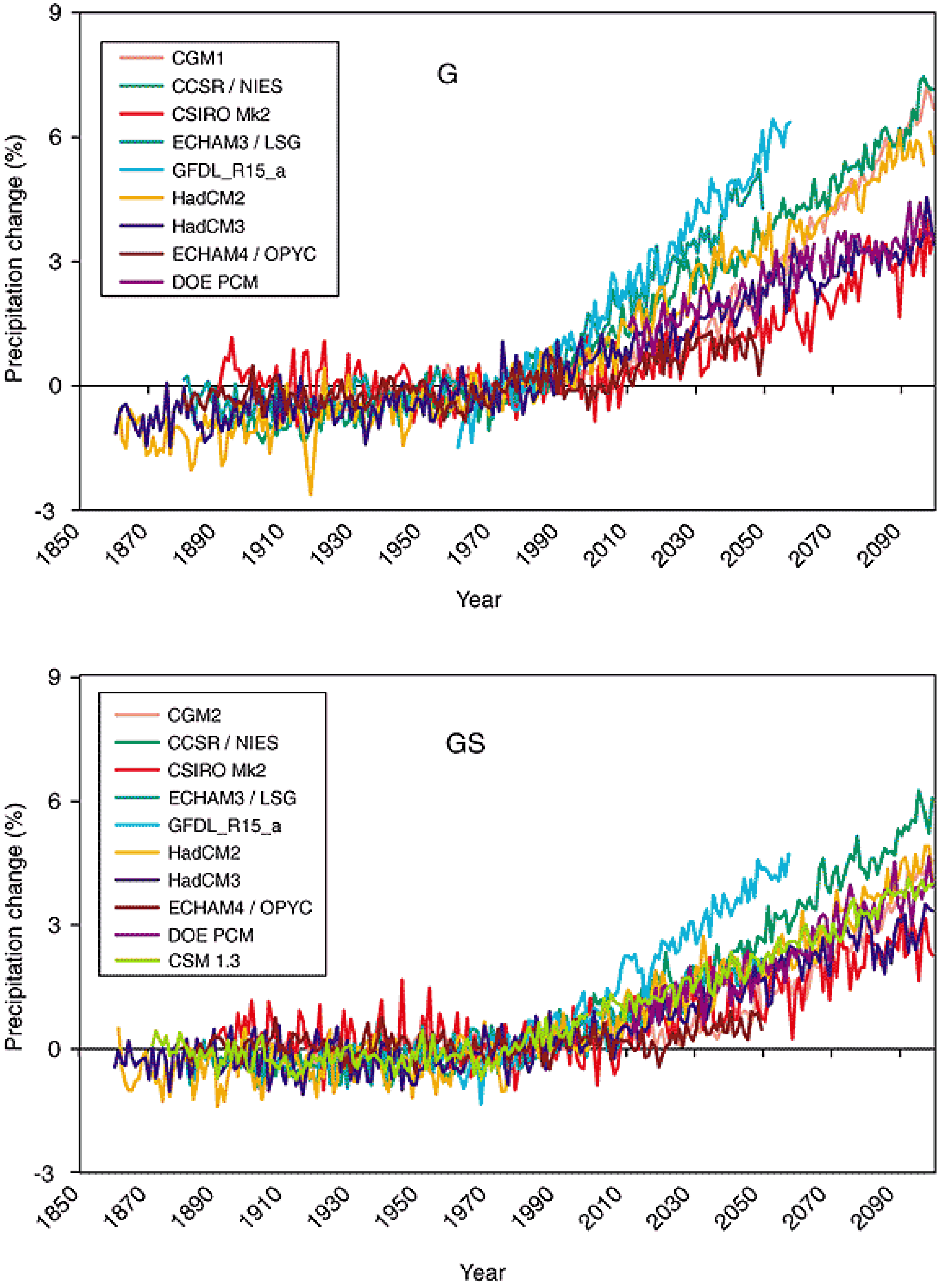

G
CGM1
CCSR / NIES
CSIRO Mk2
ECHAM3 / LSG
GFDL_R15_a
HadCM2
HadCM3
ECHAM4 / OPYC
DOE PCM
Precipitation change (%)
9
6
3
0
-3
1850
1870
1890
1910
1930
1950
1970
1990
2010
2030
2050
2070
2090
Year
GS
CGM2
CCSR / NIES
CSIRO Mk2
ECHAM3 / LSG
GFDL_R15_a
HadCM2
HadCM3
ECHAM4 / OPYC
DOE PCM
CSM 1.3
Precipitation change (%)
9
6
3
0
-3
1850
1870
1890
1910
1930
1950
1970
1990
2010
2030
2050
2070
2090
Year

Figure 9.5: *(a) The time evolution of the globally averaged temperature change relative to the years (1961 to 1990) of the DDC simulations (IS92a). G: greenhouse gas only (top), GS: greenhouse gas and sulphate aerosols (bottom). The observed temperature change (Jones, 1994) is indicated by the black line. (Unit: °C). See Table 9.1 for more information on the individual models used here. (b) The time evolution of the globally averaged precipitation change relative to the years (1961 to 1990) of the DDC simulations. GHG: greenhouse gas only (top), GS: greenhouse gas and sulphate aerosols (bottom). (Unit: %). See Table 9.1 for more information on the individual models used here.*

http://www.grida.no/climate/ipcc_tar/wg1/fig9-5.htm

XV. Lab notes on changing time steps in climate module

One of the early problems was the timing of the climate model. The lags in a 10-year time step mean that we cannot reproduce the results of climate models accurately. We therefore changed the atmospheric equation to include contemporaneous rather than one-period lag.

The tests seemed to have no technical problems. Temperature was higher as follows:

year	2005	2055	2105	2205
OLD OPT				
temp	0.71	1.671	2.534	2.703
ctax	16.4	93.24	277.93	631.61
NEW OPT				
temp	0.71	1.827	2.614	2.38
ctax	19.59	114.71	330.87	634.24

This seems analytically superior. It needs, however, to have the climate model parameters adjusted.

We begin with Twentieth-century temperature and precipitation trends in ensemble climate simulations including natural and anthropogenic forcing, Anthony J. Broccoli,1 Keith W. Dixon, Thomas L. Delworth, Thomas R. Knutson, and Ronald J. Stouffer, NOAA/Geophysical Fluid Dynamics Laboratory, Princeton University, Princeton, New Jersey, USA
Fanrong Zeng, RSIS, Princeton, New Jersey, USA. Their report is for basically the same forcings as described:

The time series of radiative forcing (relative to the preindustrial atmosphere) for the G ensemble (Figure 1a, red curve) features a slow rise from a value of _0.4 W m_2 in 1865 to _1.2 W m_2 in 1960. A more rapid rise follows thereafter, with the radiative forcing approaching 3 W m_2 by 1997.

In comparing with the Broccoli et al model, that model with the forcings leads to an increase of of about 1.05 oC over the period. This is consistent with the earlier calibration.

We therefore recalibrated as follows:

1. We changed the equilibrium T* to 3o C.
2. We then changed the atmos/ocean coefficient to calibrate with both the doubling and the CFDL scenarios. The parameter of .215 yielded a good match, a little low for the CO2 doubling and a little high for the GFDL scenario:

Difference from 1880-1920			
1880-1920		2000.000	
	0.245	1.296	1.051
CO2 doubling from IPCC			
		1.784	
Speed of adjustment parameter for atmospheric temperature		0.215	

Comparing to the results pre-adjustment, we get the following. The adjustments tend to make the temp path a little higher (because simultaneous), increasing the carbon tax by about 10 percent to $20.

OPTIMAL				
year	2005	2055	2105	2205
Carbon tax				
Pre carbon and climate adjusted	17.08	96.27	284.28	529.24
Latest	19.86	116.52	337.44	633.93
Temp				
Pre carbon and climate adjusted	0.710	1.723	2.636	2.724
Latest	0.710	1.834	2.643	2.420
Concentrations				
Pre carbon and climate adjusted	787.0	1055.2	1291.2	1094.1
Latest	787.0	1020.2	1218.9	990.3
BASE				
Temp				
Pre carbon and climate adjusted	0.710	1.762	2.891	4.777
Latest	0.710	1.903	3.013	4.824
Concentrations				
Pre carbon and climate adjusted	787.0	1086.7	1486.9	2406.1
Latest	787.0	1055.1	1422.9	2279.1

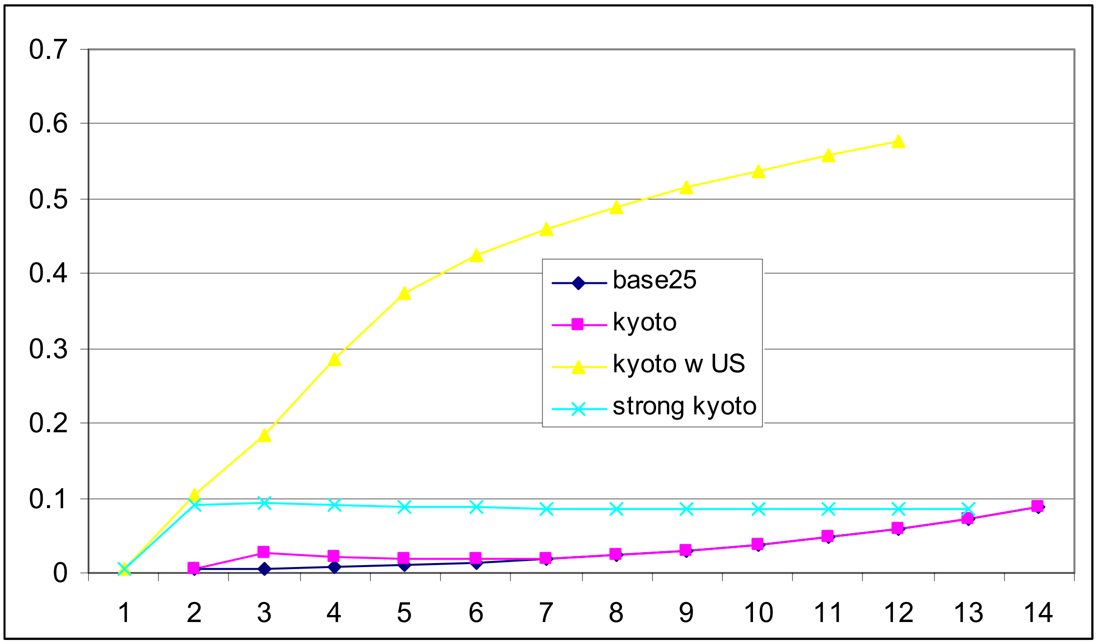
XV. Miscellaneous Modeling Details

a. Initial Period Time Step

The initial time step is a complication because it include largely the historical period. However, we still want to include it. We assume that the period includes the first two years of the KP, which we estimate to have a global reduction rate of 3 percent for the last two years, which is a decadal reduction rate of 0.3 percent. (macro_kyoto_032107.xls, sheet "Kyoto", lines AZ:BA72). We raise this to 0.5 percent to capture anticipatory effects and non-participants. Note that this is the same in all runs. It means, however, that first-period prices are non-informative, and second-period ones are the useful ones.

b. Kyoto Runs

The Kyoto runs have a computational problem because the control rates are so low. The Hotelling rents and carbon taxes cross relatively early, so the Kyoto "def" files need to ensure that the Hotelling rent takes over at the crossover point. The following shows an example for the Kyoto without US (from sheet "Economics"):



c.

XVI. Notes on uncertain parameter

a. The Monte Carlo estimates

We have undertaken a preliminary Monte Carlo estimate to determine if a larger sample would produce higher variances or evidence of fat tails. For this purpose, we took the random 100 runs and estimated a "response surface" as a function of the random variables. The estimated equation was the following for a simple and for a polynomial function:

```
Dependent Variable: MARGY
Method: Least Squares
Date: 07/18/07   Time: 16:41
Sample (adjusted): 1 100
Included observations: 100 after adjustments
==========================================================================
        Variable        Coefficient    Std. Error    t-Statistic    Prob.
==========================================================================
       A_CARBCYC          3298.726      285.3171      11.56161     0.0000
       A_DAMCOEF          12678.03      5763.093      2.199865     0.0303
        A_DECARB         -1044.854      382.2601     -2.733360     0.0075
        A_FOSSIL          0.008743      0.006374      1.371721     0.1735
         A_PBACK          31.11352      16.26451      1.912970     0.0589
           A_POP          0.045886      0.003963      11.57901     0.0000
        A_T2XCO2          9.297585      6.819875      1.363307     0.1761
          A_TFPG         -796.7031      191.9248     -4.151120     0.0001
==========================================================================
R-squared                 0.043463    Mean dependent var           1177.421
Adjusted R-squared       -0.029317    S.D. dependent var           74.31845
S.E. of regression        75.39998    Akaike info criterion        11.56011
Sum squared resid         523034.4    Schwarz criterion            11.76852
Log likelihood           -570.0055    Durbin-Watson stat           2.028405
==========================================================================
```

```
Dependent Variable: MARGY
Method: Least Squares
Date: 07/18/07   Time: 16:40
Sample (adjusted): 1 100
Included observations: 100 after adjustments
==========================================================================
        Variable        Coefficient    Std. Error    t-Statistic    Prob.
==========================================================================
       A_CARBCYC          9336.260      372.5297      25.06179     0.0000
```

A_DAMCOEF	-1873.304	3373.255	-0.555340	0.5802
A_DECARB	-566.9492	328.9890	-1.723308	0.0886
A_FOSSIL	0.020930	0.008258	2.534523	0.0132
A_PBACK	9.118749	11.91313	0.765437	0.4462
A_POP	0.041152	0.004502	9.140391	0.0000
A_T2XCO2	4.457665	5.388065	0.827322	0.4105
A_TFPG	-1288.599	260.3073	-4.950298	0.0000
A_CARBCYC^2	-24526.64	1040.204	-23.57867	0.0000
A_DAMCOEF^2	127666.9	563876.8	0.226409	0.8214
A_DECARB^2	-2896.838	2211.598	-1.309840	0.1939
A_FOSSIL^2	-1.71E-06	6.72E-07	-2.552498	0.0125
A_PBACK^2	-3.611497	4.924094	-0.733434	0.4654
A_POP^2	-5.81E-07	2.62E-07	-2.216236	0.0294
A_T2XCO2^2	0.124151	0.882171	0.140733	0.8884
A_TFPG^2	2867.141	3049.376	0.940239	0.3499
A_TFPG^3	-4023.028	10442.88	-0.385241	0.7011
A_T2XCO2*A_TFPG	-65.35669	29.68070	-2.201993	0.0305

R-squared	0.976919	Mean dependent var	1177.421
Adjusted R-squared	0.972134	S.D. dependent var	74.31845
S.E. of regression	12.40599	Akaike info criterion	8.035784
Sum squared resid	12620.50	Schwarz criterion	8.504715
Log likelihood	-383.7892	Durbin-Watson stat	2.104314

We then predicted the major variables using a sample of 10,000 realizations of the random variables. The following shows the major statistics for the SCC, temperature at 2205, and the marginal utility of consumption:

	MARGYF	MARGYF_QUAD	SCC1F	SCC1F_QUAD	TEMP21F	TEMP21F_QUAD
Mean	1178.760	1179.265	26.70280	26.57500	5.033002	4.967514
Median	1179.226	1180.559	26.71182	27.84185	5.043746	5.004213
Maximum	1447.814	1449.938	85.37925	85.83153	12.45189	11.38508
Minimum	899.5732	874.3436	-43.92759	-59.45535	-2.033592	-2.724209
Std. Dev.	73.51238	74.66149	17.13403	17.42335	1.823906	1.876188
Skewness	-0.014920	-0.049511	-0.016256	-0.488811	-0.017054	-0.163506
Kurtosis	2.970143	3.067010	3.005680	3.633053	3.069014	3.160268
Jarque-Bera	0.742441	5.956634	0.453865	565.2080	2.469309	55.25962
Probability	0.689892	0.050878	0.796974	0.000000	0.290935	0.000000
Sum	11787603	11792648	267028.0	265750.0	50330.02	49675.14
Sum Sq. Dev.	54035302	55737812	2935455.	3035428.	33263.01	35197.29
Observations	10000	10000	10000	10000	10000	10000

Most of the specifications are close to normal, although two show high Jarque-Bera statistics, indicating significant kurtosis. There is no indication of an increasing variance as the sample size increases. The standard deviations are virtually identical for a sample of 100, 1000, and 10000. A test of the coefficient of the Pareto distribution for the largest 1000 values of SCC gives a coefficient of -9, and similar results are found for the other variables. The tail has finite variance for a coefficient > -2.

b. Estimation of uncertain variables

September 4, 2007

We recalculated the parameter calculation for the temperature-sensitivity coefficient and refined the description, adding the new description in the text.

July 18, 2007

We have incorporated the revisions of delta.8 runs in the uncertainty analysis. We first created the new file (in C:\Major\Res-clim\DICE\DICE_delta8_unc\Base_newparams). We then verified that it gave the same results as the last runs. Note that we widened the upper and lower limits in the new runs.

The next step was to perform the "sigma tests." These examine the impact of single variables on the outcome, with the variable going from -6sigma to +6sigma of its value.

June 7, 2007

The estimates of the uncertain parameters were drawn from the existing literature as well as on the author's judgment. The starting point was the earlier uncertainty analysis in the study, *Managing the Global Commons,* Chapter 6. The top uncertain variables in terms of contributing to overall uncertainty were the following:

- decline of rate of growth of population (asymptotic population)
- decline of rate of growth of productivity (growth in total factor productivity)
- pure rate of social time preference (same)
- rate of decline in the CO2-output ratio (same)
- intercept of the damage function (same)
- climate-GHG sensitivity coefficient (same)

- intercept of the mitigation-cost function (price of backstop technology)
- rate of atmospheric retention of CO2 (transfer coefficient)

We have added in parentheses the corresponding parameter in DICE-2007 model or "same" where the concept is the same. For the current runs, we omit the pure rate of time preference because that is a preference rather than a technological parameter. We have added an additional parameter, the resources of fossil fuels, which was not in the original DICE model.

There are multiple sources for the estimates of the uncertainty of the parameters. One comparison is from DICE 1994, and the following shows the coefficients of variation of the parameters which are easily compared. Additionally, a number of alternative estimates have been used for comparison. We have found no useful comparative studies of the uncertainty of damages. In general, the calibration is to the coefficient of variation in the different sources.

It should be emphasized that the uncertainty estimates are themselves very uncertain. Few of them have been calibrated to actual historical data, and some, like future productivity growth, are inherently unknowable.

Variable	Definition	Units	Mean	Standard deviation	Coefficient of variation, DICE-2007	Coefficient of variation, DICE-1994	Alternative estimates	
ga0	Rate of growth of total factor productivity	Per year	0.9200	0.4000	0.435	0.680	0.19-0.54	(b, g)
gsigma	Rate of decarbonization	Per year	0.7000	0.2000	0.286	0.650	0.250	(b)
t2xco2	Equilibrium temperature sensitivity coefficient	oC per CO2 doubling	3.0000	1.1100	0.370	0.354	0.372	(i)
a2	Damage parameter (intercept of damage equation)	Fraction of global output	0.00278	0.00130	0.467	0.846		
pback	Price of backstop technology	$ per ton C replaced	1,170	468	0.400	0.551	0.300	(b)
popasym	Asymtotic global population	Millions	8,600	1,892	0.220		0.224	(d)
b12	Transfer coefficient in carbon cycle	Per decade	0.189	0.017	0.090		0.090	(e, f)
fosslim	Total resources of fossil fuels	Billions tons carbon	6,000	1,200	0.200		0.445	(g)

Source
(a) Uncertainty Analysis of Climate Change and Policy Response, M. Webster, C. Forest, J. Reilly, M. Babiker, D. Kicklighter, M. Mayer, R. Prinn, M. Sarofim, A. Sokolov, P. Stone and C. Wang, Report No. 95, December 2002,
(b) Mort D. Webster, Uncertainty in Future Carbon Emissions: A Preliminary Exploration, Massachusetts Institute of Technology,Joint Program on the Science and Policy of Global Change, Report #30, November 1997
(d) IIASA stochastic population projections, personal communication, Wolfgang Lutz, May 2007.
(e) The uncertainty in DICE-2007 is set on the basis of the uncertainty generated by the MAGICC simulations, which have a standard deviation of approximately 40 ppm in 2100 for the A1F1 scenario.
(f) From the cross-sectional uncertainties of the model comparison in Riahi, Keywan, Arnulf Gruebler, and Nebojsa Nakicenovic [2007]. “Scenarios of long-term socio-economic and environmental development under climate stabilization,” Technological Forecasting & Social Change, forthcoming.
(g) Nordhaus and Yohe, "Future Paths of Energy and Carbon Dioxide Emissions," NRC, 1982.
(h) H. Rogner, Ann. Rev. Energy and Env. 22, p. 249. But this is the range with and without speculative resources, which is interpreted as 4 sigma.
(i) IPCC, Fourth Assessment Report, Science, Chapter 10. For this estimate, we take the average of results for the 18 AOGCMs and the estimates constrained by climatology, which yield a 5-95 percent range of 3.63 oC. This is