Projections of Sea Level Rise (SLR)

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The 2010 RICE model adds a module to project future sea-level rise (SLR) in coming centuries. It is recognized that this is a very dynamic area of research, that there are major discrepancies among models, and that the dynamics of the large ice sheets is extremely complex. However, omitting SLR from a model of the economics of climate change is missing one of the major concerns. The methodology of the modeling is to use the estimates in the IPCC Fourth Assessment Report (AR4).[1] This report is the latest consensus and contains a largely internally consistent set of assumptions and presentations. While it is recognized that the science is changing quickly, we have generally preferred to remain with consensus views rather than attempt to synthesize a moving set of revisions. We have also compared the results with Rahmstorf's "semi-empirical" method.

A. The Model

We describe here the sea-level rise module and its derivation. The modeling contains the four major processes that affect sea level rise.

1. Thermal expansion

Water density is a function of both temperature and pressure. This implies that estimating the impact of global warming on thermal expansion requires three-dimensional modeling. The most useful approach for our purposes is the Earth System Models of Intermediate Complexity (EMICs). Modeling runs including SLR over the period to 3000 estimate an equilibrium thermal expansion 0.2 to 0.6 m per °C for their final steady state (year 3000) relative to 2000. Based on the model averages in Figure 10.34, p. 823, we assume that the long-run year expansion is 0.5 m per °C.

We assume that the adjustment to the long run is a first-order adjustment process. These yield the following equations:

[1] References to the Fourth Assessment Report ("AR4") are by page numbers. The full citation is Intergovernmental Panel on Climate Change, *Climate Change 2007: The Physical Science Basis,* Contribution of Working Group I to the Fourth Assessment Report of the IPCC, available online at http://ipcc-wg1.ucar.edu/wg1/wg1-report.html.

$$(1) \quad SLR^* = 0.5\, T_t$$

$$(2) \quad \Delta SLR_t = \lambda_{SLR}[SLR^* - SLR_{t-1}]$$

$$(3) \quad \lambda_{SLR} = 0.02 \ \ per\ decade$$

The adjustment parameter in (2) is calculated from the EMICs and the AOGCMs in AR4. The calculation is available in the data spreadsheet. The errors in the process are minimized for an adjustment parameter λ = 0.02 per decade. The model tends to underpredict in the short run and overpredict in the long run, but it captures the projections of the models reasonably accurately for our purposes.

2. <u>Glaciers and Small Ice Caps</u>

Glaciers and small ice caps are a second source of SLR. Estimates of the total SLR equivalent vary widely (see AR4, Table 4.1). We take the estimate of the process from AR4's discussion, which provides a central estimate of 0.26 m of SLR equivalent and a melt rate of 0.0008 m per year per °C relative to global T of -1 °C from 2000. The overall results are not significantly affected by this contribution.

$$(4) \quad \Delta SLRe_t^{Glac} \ = \ 0.08\, mm\, yr^{-1}\ {}^{o}C^{-1}$$

3. <u>Greenland Ice Sheet</u>

The dynamics of the Greenland Ice Sheet (GIS) is extremely complex and incompletely understood. Large scale AOGCMs are thought not to accurately represent the processes of large ice sheets because of the poor resolution, difficulties in modeling the steep slopes where the action occurs, and smooth model topography (AR4, p. 816). The "standard models" used in recent years have been cast into doubt by recent observations, but to date no modeling results have successfully captured all the complexities of the GIS. There are large discrepancies among the different models as to the sensitivity of the GIS to warming, with differences being a factor of close to 10 between the extremes.

The present study relies primarily upon the coupled simulation of Ridley et al. 2005. This study finds a SLR contribution of 5.5 mm per year over the first 300 years for a 4xCO2 simulation. This appears to be consistent with a 6 °C global warming. This calibrates to a melt rate of 0.1113 mm per year per °C for global temperatures above 1 °C. It is assumed that the melt rate declines as the volume declines.

For the present study, we assume the equilibrium volume is a function of equilibrium temperature. For simplicity, we assume that the current volume is the equilibrium for T < 1 °C above preindustrial levels; that the equilibrium volume declines linearly from the current to zero at T = 3.5 °C; and that the ice sheet has zero equilibrium mass for higher equilibrium temperatures. Based on the study of Ridley et al. we assume that net volume decrease is 0.8 mm of SLR per year per °C times the ratio of the volume of the ice sheet to its original volume.[2] We write the dynamics as follows:

$$
\begin{aligned}
(5)\quad SLRe^{GIS}* &= 0 \ \ meters\ SLRe\ for\ T \leq 1\ ^{o}C \\
&= [0.4\,(T - 1)]\, 7.3\ \ SLRe\ for\ 1\ \ ^{o}C \geq T \geq 3.5\ ^{o}C \\
&= 7.3\, SLRe\ for\ T \geq 3.5\ ^{o}C
\end{aligned}
$$

$$
(6)\quad SLRe_{t}^{GIS} = SLRe_{t-1}^{GIS} + \lambda_{SLR}^{GIS}\, [SLRe^{GIS} * - SLRe_{t-1}^{GIS}\,]
$$

$$
(7)\quad \lambda_{SLR}^{GIS} = \varphi_1 T + \varphi_2 T^2 \qquad
\begin{aligned}
&= 0.4\ mm\ yr^{-1}\ ^{o}C^{-1} \times SLRe_{t-1}^{GIS}\ \ at\ \ T = 3^{o}C \\
&= 0.8\ mm\ yr^{-1}\ ^{o}C^{-1} \times SLRe_{t-1}^{GIS}\ \ at\ \ T = 6^{o}C
\end{aligned}
$$

Note that this approach assumes that there is no hysteresis in the dynamics of the GIS. This issue is postponed for future study.

4. Antarctic Ice Sheet

The Antarctic Ice Sheet (AIS) and particularly the West Antarctic Ice Sheet (WAIS) pose difficult modeling issues for the long run. In the short run of a century of so, most models suggest that the AIS will be contributing negatively to SLR. The central tendency of models suggests a contribution of approximately -1 mm per year to sea level. However, this negative contribution is potentially offset by instabilities, particularly arising from the WAIS.

The potential instability of the WAIS has been recognized for many years. The stakes are large, because the WAIS contains approximately 5 meters of SLRe. However, there are apparently no grounds for making firm projections. The AR4 concluded its discussion, "Because the available models do not include all relevant processes, there is much uncertainty and no consensus about what dynamical changes could occur in the Antarctic Ice Sheet…" (p. 831) They suggest that an upper limit of discharge is 10 mm per year, while a second upper bound is set at 2.5 mm per year. (p. 831)

[2] See the discussion in AR4, Chapter 10.

We recognize that modeling in this area is highly conjectural. We assume that that the melting point for the WAIS is a global temperature increase of 3 °C from pre-industrial levels. We further assume that the discharge increases linearly between 3 °C and 6 °C, with a maximum discharge rate of 2.5 mm SLRe per year at 6 °C. This implies that at the maximum discharge rate the WAIS would be fully discharged after 500 years. The equations of the system are therefore:

$$(8) \quad SLRe_t^{AIS} = SLRe_{t-1}^{AIS} + \lambda_{SLR}^{AIS}$$

$$(9) \quad \lambda_{SLR}^{AIS} \begin{cases} = -0.10 \text{ } mm \text{ } yr^{-1} \text{ for } T \leq 3 \text{ }^{o}C \\ = -0.10 + 0.833 \text{ } mm \text{ } yr^{-1} \times (T - 3)^{o}C \text{ for } 3 \text{ }^{o}C \leq T \leq 6 \text{ }^{o}C \\ = 2.4 \text{ } mm \text{ } yr^{-1} \text{ for } T£ \text{ } 3 \text{ }^{o}C + 5 \text{ } SLRe \text{ for } T \geq \text{ }^{o}C \end{cases}$$

B. Results

We next show the preliminary results. For these runs, we use the RICE-2010 model baseline (with no policy restrictions on GHG emissions). Figure 1 shows the projected temperatures used in the baseline. Note that the model assumes that a competitive zero-carbon energy source becomes available in 2250, so carbon emissions decline quickly after that point. Projected temperatures for these simulations peak at around 5½ °C and stabilize at 5 °C.

In the short run until 2100, the estimated SLR is about 0.20 meters above the 2000 level. This is close to the middle of the model estimates for scenario A1B, which resembles closely the temperature profile shown in Figure 1. The other major comparison is with the EMICs, for which several models used a constant CO2 concentrations at 700 ppm to project thermal expansion. These show a lower temperature path than the RICE model. The central tendency of EMICs for this path is for a SLR of about 1.1 meter by 3000, whereas the RICE module estimates a SLR from thermal expansion of 2 meters with the higher temperature trajectory. Using a temperature path of 3.2 °C after 2100 yields a thermal expansion of 1.4 m by 3000, which is slightly above the calculations of the EMICs.

The other major point that is emphasized in these projections is the importance of the large ice sheets, particularly in the longer run. Our order-of-magnitude estimates indicate that thermal expansion contributes between one-third and one-fourth of the long-run SLR (beyond 2300). Clearly, much more emphasis is needed in developing these modules. Additionally, further work is needed to capture the potential hysteresis in the large ice sheets, which may be particularly important for the GIS.

It should be emphasized that the SLR module developed for the RICE model is mainly to be used for purposes of linking the different parts of the model (economics, climate change, impacts, and policy). Other, well-developed geophysical models will be much better suited to get the precise details of the SLR correctly calculated. The spirit of the estimates is to get a "one-digit approximation" to the scientific consensus as published in IPCC reviews. It is also sufficiently detailed that new estimates can be easily incorporated in the model.

C. Comparison with Rahmstorf's Semi-Empirical Method

A useful comparison is the Stefan Rahmstorf's "semi-empirical method" for estimating the contribution of warming to SLR.[3] The approach is the following (using slightly different notation).

(1) $SL^* = f(T)$

where $SL^* =$ the equilibrium eustatic sea level, T is global mean temperature, and $f(T)$ is the (possibly non-linear) relationship. We assume that the relationship can be represented by the simple first-order process:

(2) $SL_t - SL_{t-1} = \lambda[f(T_t) - SL_{t-1}]$.

For the estimates, we assume that the time step is one year.

We first estimate the equation provided in Rahmsdorf, which is

(3) $SL_t - SL_{t-1} = \alpha_0 + \alpha_1 T_t + \varepsilon_t$

Note that Rahmsdorf does not explicitly model this as a stochastic process and that the lagged term in sea level is omitted.

For the estimates, we assume that the time step is one year.

We first estimate the equation similar to that discussed in Rahmstorf (the estimated equation is not provided in either article but is available in the Matlab code). The estimate is very similar to that in Rahmstorf (reported as 3.4 mm/year per °C). However, the R (= .092) is inconsistent with that reported in Rahmstorf (R = .88), and the coefficient is statistically insignificant, as shown below:

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[3] See Stefan Rahmstorf, "Sea-Level Rise: A Semi-Empirical Approach to Projecting Future Science 315, 368 (2007); Martin Vermeera and Stefan Rahmstorf, "Global sea level linked to global temperature," *PNAS,* December 22, 2009, vol. 106, no. 51, 21527–21532.

Dependent Variable: D(GSL)
Method: Least Squares
Sample: 1850 2009
Included observations: 160

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.920803	0.974483	1.971099	0.0505
TAVNEW	3.632790	3.126313	1.162005	0.2470

R-squared	0.008474	Mean dependent var	1.554543
Adjusted R-squared	0.002198	S.D. dependent var	11.67658
S.E. of regression	11.66374	Akaike info criterion	7.763269
Sum squared resid	21494.78	Schwarz criterion	7.801708
Log likelihood	-619.0615	Hannan-Quinn criter.	7.778878
F-statistic	1.350255	Durbin-Watson stat	1.849217
Prob(F-statistic)	0.246986		

The source of the difference is because of slightly different data (the series here has been extended through 2009). The major difference is that Rahmstorf and Vermeera and Rahmstorf have applied a smoothing program to remove the high frequency "noise" from the data. This is not generally warranted without attention to the source of the error.

As an example, I used 15-year moving averages of the two series. This produced the following, which has highly significant coefficients and a higher R2.

Dependent Variable: D(MAGSL15)
Method: Least Squares
Date: 02/09/10 Time: 22:16
Sample (adjusted): 1864 2009
Included observations: 146 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.079245	0.133612	15.56180	0.0000
MATEMP15	2.747826	0.522747	5.256512	0.0000

R-squared	0.160990	Mean dependent var	1.759441
Adjusted R-squared	0.155164	S.D. dependent var	1.563795
S.E. of regression	1.437361	Akaike info criterion	3.577099
Sum squared resid	297.5051	Schwarz criterion	3.617970
Log likelihood	-259.1282	Hannan-Quinn criter.	3.593706
F-statistic	27.63092	Durbin-Watson stat	0.521252
Prob(F-statistic)	0.000001		

However, this is spurious as the underlying process is not an AR or MA process:

Dependent Variable: D(GSL)
Method: Least Squares
Date: 02/09/10 Time: 22:21
Sample (adjusted): 1850 2009
Included observations: 160 after adjustments
Convergence achieved after 7 iterations
MA Backcast: 1849

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.917422	1.094107	1.752499	0.0816
TAVNEW	3.595246	3.478194	1.033653	0.3029
MA(1)	0.127189	0.079303	1.603832	0.1108

R-squared	0.018361	Mean dependent var	1.554543
Adjusted R-squared	0.005856	S.D. dependent var	11.67658
S.E. of regression	11.64234	Akaike info criterion	7.765746
Sum squared resid	21280.42	Schwarz criterion	7.823406
Log likelihood	-618.2597	Hannan-Quinn criter.	7.789160
F-statistic	1.468327	Durbin-Watson stat	2.047793
Prob(F-statistic)	0.233456		

Inverted MA Roots	-.13

I compared the results by doing three different sets of tests. First, I used a n-period rather than a 1-period difference to the underlying equation, for differences up to 20 years. Second, I did the same and applied second-order AR and eighth-order MA error correction to the equation. Finally, I lagged the temperature series one year. Figure 5 shows the coefficients and error bands for the 60 coefficient estimates. The estimated coefficient of the base Rahmstorf equation shown above is higher than the rest of the coefficients, with most of the coefficients in the 2-3 mm per °C range.

We then took the 60 different specifications, forecast sea level to 2200, and compared with the RICE model projections, using the RICE temperature projection. The results are shown in Figure 6.

The RICE model projection is in the middle of the pack of alternative specifications of the different Rahmstorf specifications. Table 1 shows the RICE, base Rahmstorf, and average Rahmstorf. Note that in all cases, these are significantly above the IPCC projections in AR4.

Year	RICE	Rahmstorf semi-empirical	
		Base	Average of 60
	[mm above 1900]		
2000	139	142	143
2100	727	1,080	841
2200	2,162	2,946	2,166

Table 1. Comparison of different estimates

The tests of the RICE model SLR module in comparison with the Rahmstorf semi-empirical method suggest the following. First, the RICE model SLR is less rapid than the baseline Rahmstorf projection. Secondly, however, the Rahmstorf model is sensitive to alternative specifications, and with unstable and sometimes insignificant coefficients. The RICE model projections are consistent with the average of the alternative specifications tested here.

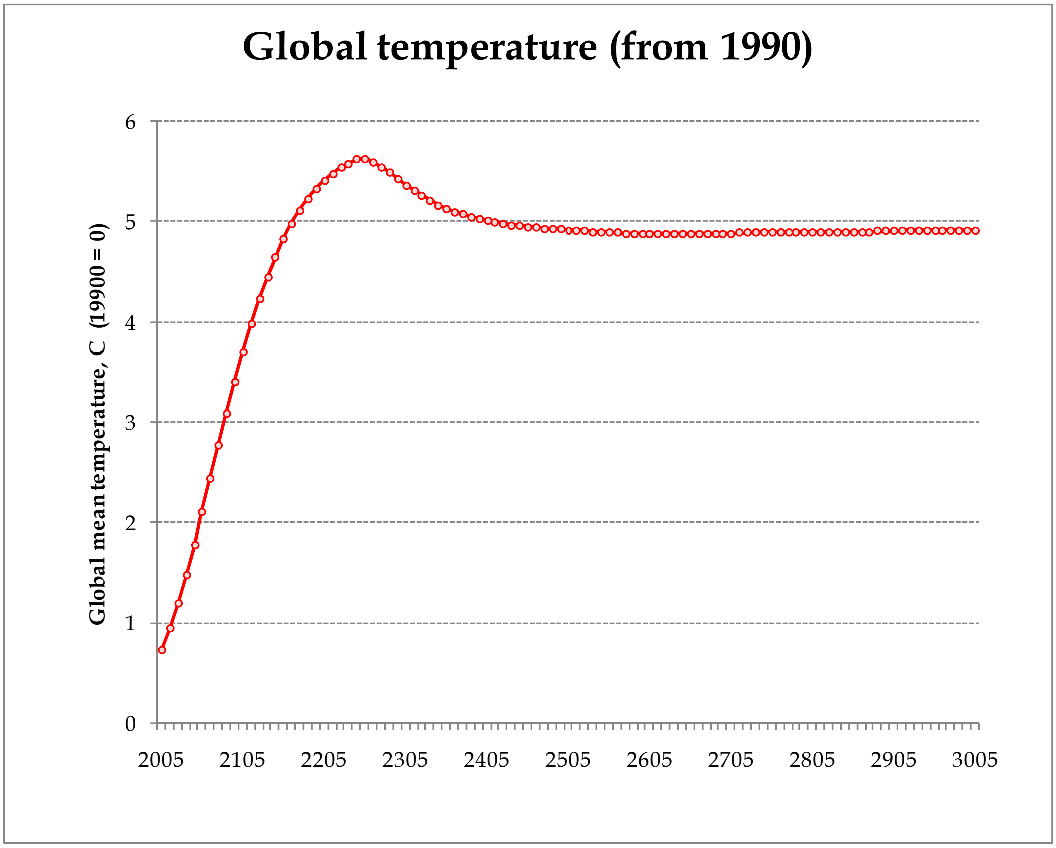

Figure 1. Projected Global Temperature in Runs

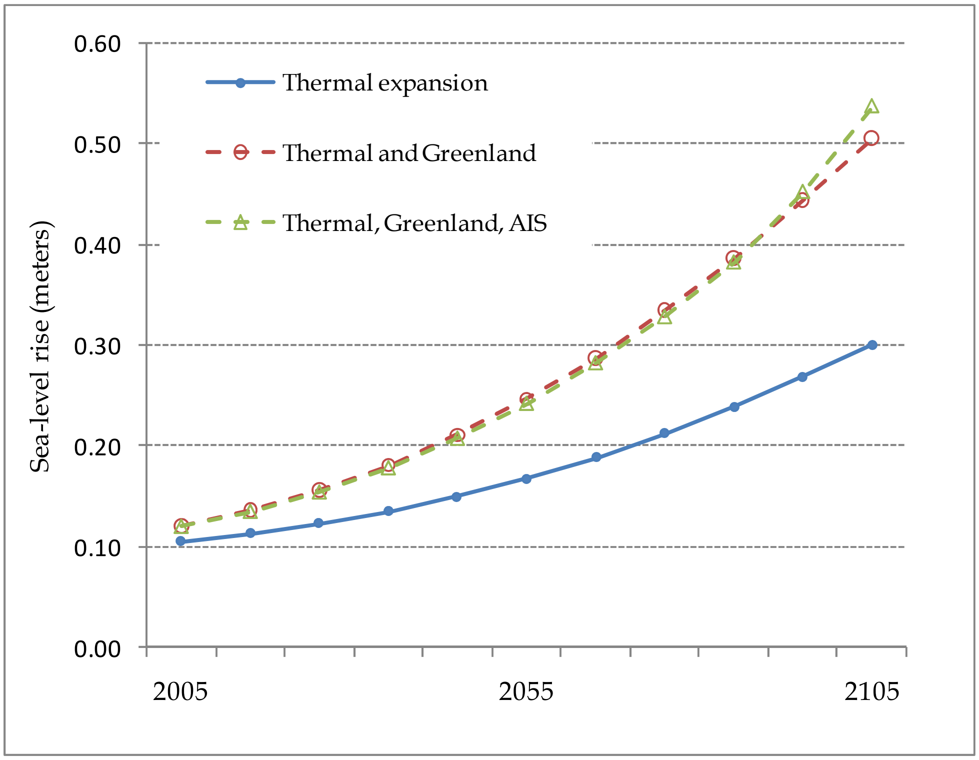

Figure 2. Projected Short Run SLR

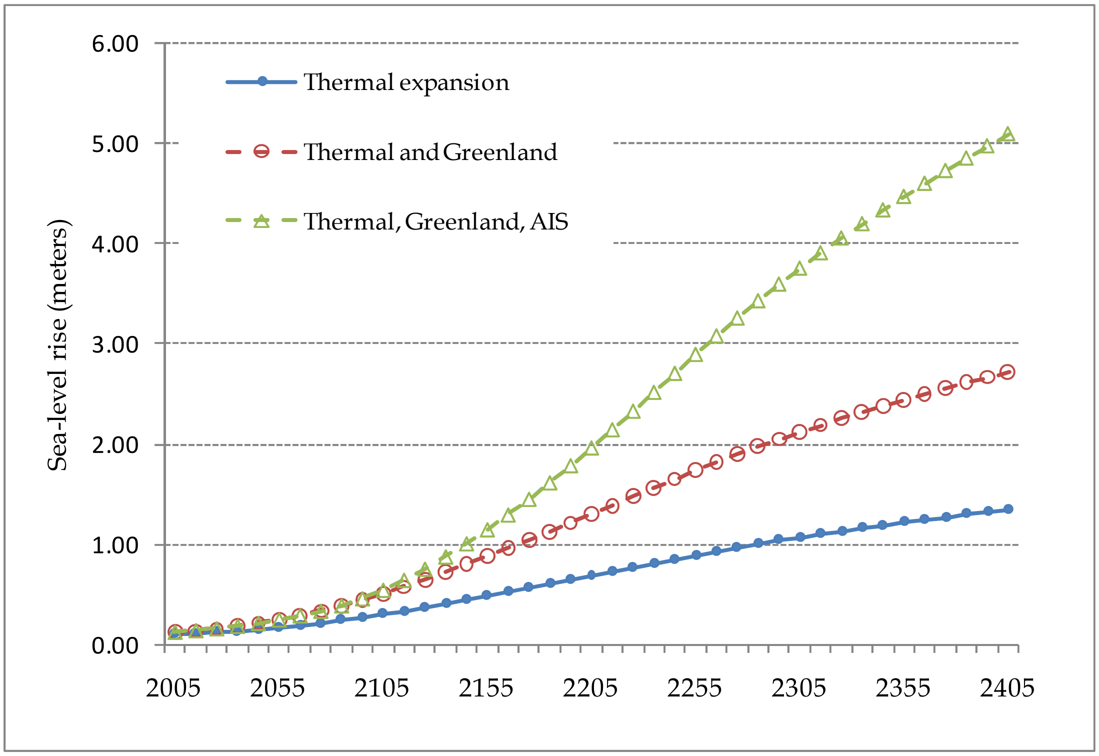

Figure 3. Projected Medium Run SLR

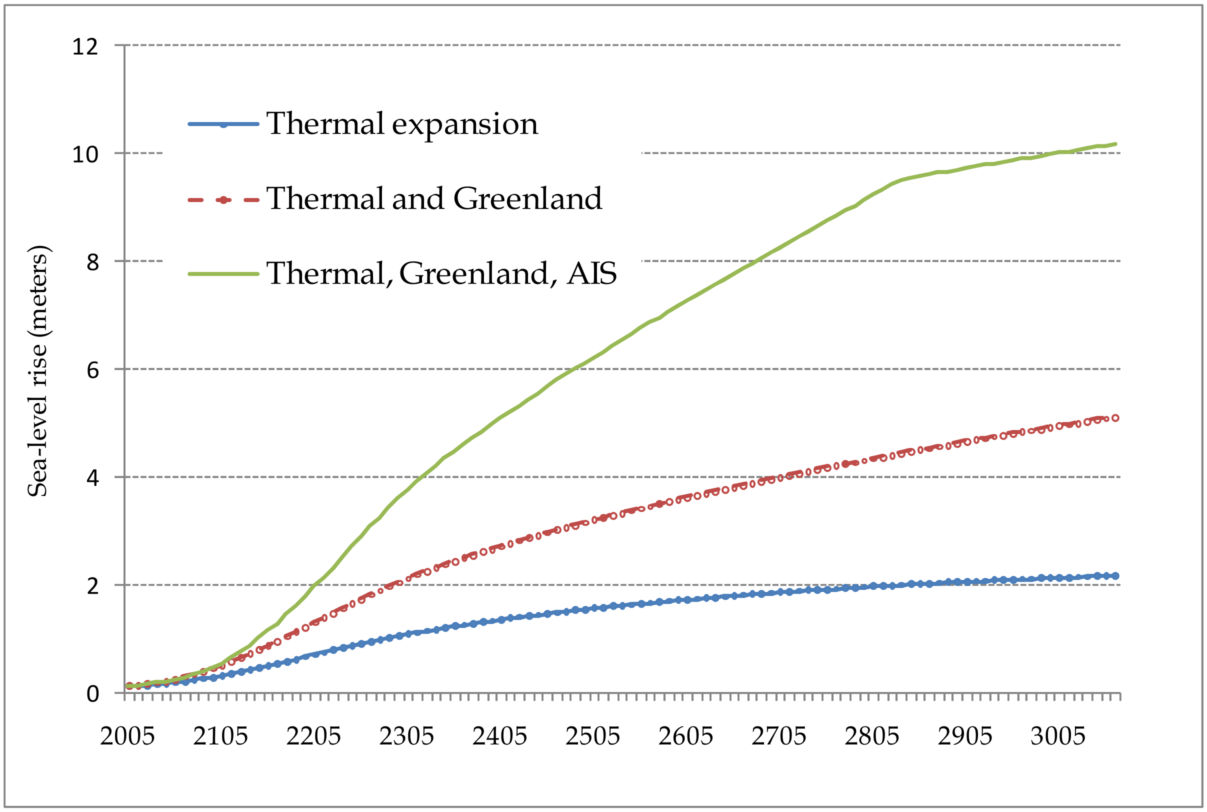

Figure 4. Projected Very Long Run SLR

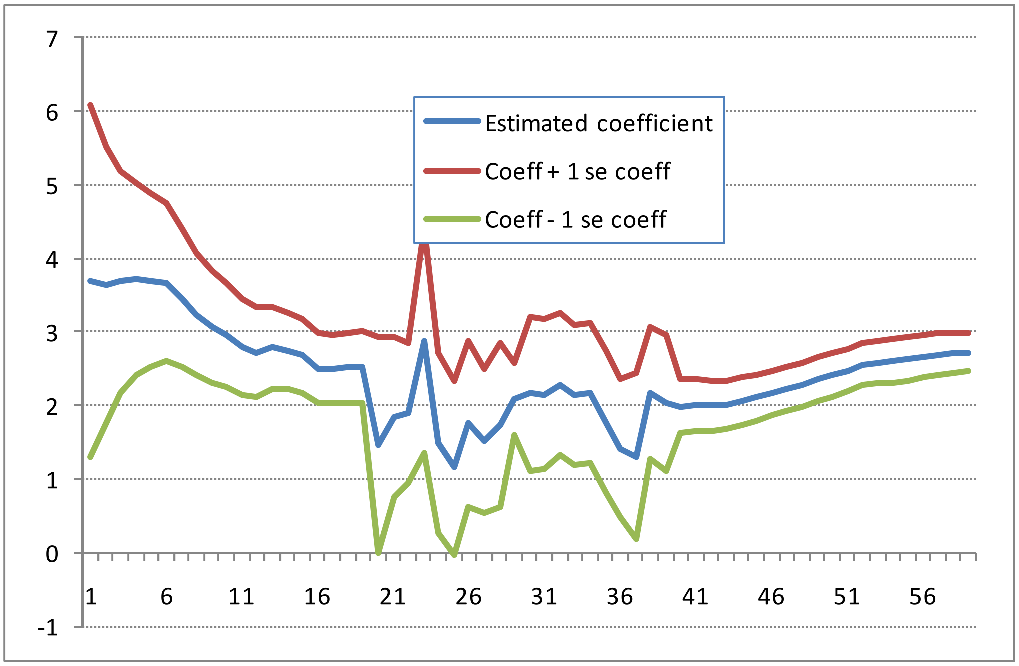

Figure 5. Alternative coefficients for sea-level rise equation

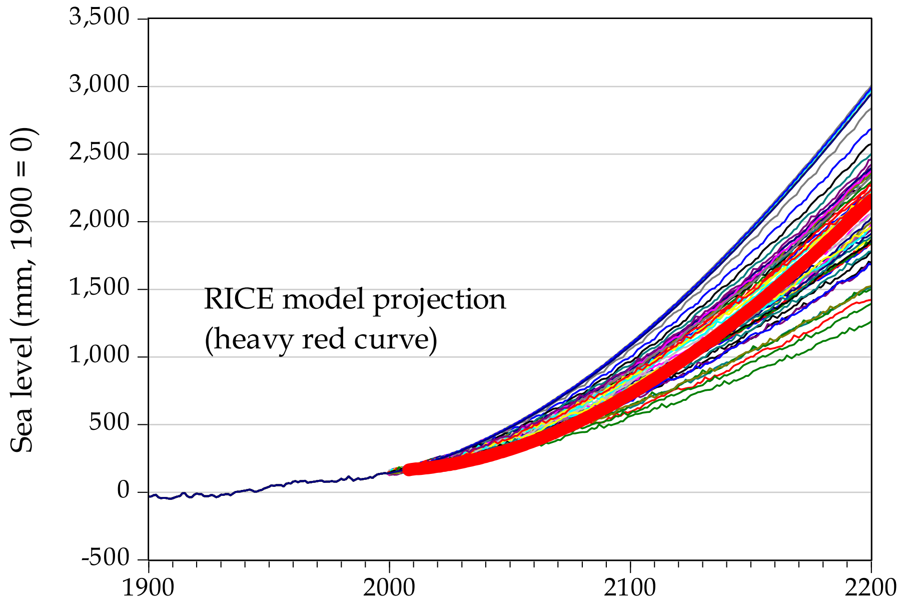

Figure 6. Alternative projections of SLR